John Shietz

BELL TELEPHONE LABORATORIES
INCORPORATED

COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE - Realization of FEN RC Input Networks

MM 67 - 5316 - 3

DATE— May 15, 1967

CASE CHARGED— 39095-19

FILING CASES— 38763-17

AUTHOR— D. G. Marsh

FILING SUBJECTS— Active RC Filters

ABSTRACT

RC networks to be used at the input of a Frequency Emphasizing Network for the design of an active filter building block are considered. Curves are given to facilitate the design of low-pass, band-pass, and high-pass filters as well as resonators.

BELL TELEPHONE LABORATORIES
INCORPORATED

MM— 67-5316-3

DISTRIBUTION
(REFER G.E.I. 13.9-3)

COMPLETE MEMORANDUM TO

CORRESPONDENCE FILES:

OFFICIAL FILE COPY (FORM E-7770) — PLUS ONE WHITE COPY FOR EACH ADDITIONAL CASE REFERENCED

REPRODUCIBLE COPY (FORM E-1328)

REFERENCE COPIES (10)

PATENT DIVISION (2) IF MEMORANDUM HAS PATENT SIGNIFICANCE

W. M. Bacon - HO
W. L. Brune - WL
G. K. Burns - HO
H. O. Burton - HO
A. Busala - IN
R. E. Cardwell - HO
J. R. Davey - HO
W. H. D'Zio - HO
A. C. Ekvall - WL
B. E. Epting - GR
D. Feldman - WH
W. O. Fleckenstein - HO
R. D. Fracassi - HO
F. E. Froehlich - HO
A. M. Gerrish - HO
D. Hirsch - HO
H. J. Jones - WL
N. Knapp, Jr. - HO
M. C. Kolibaba - HO
J. T. Koo - AL
E. R. Kretzmer - HO
J. J. Lang - MV
W. J. Lawless - HO
G. H. Lovell - WL
R. W. Lucky - HO
G. Malek - HO
D. G. Medill - HO
G. S. Moschytz - HO
C. R. Moster - HO
W. H. Orr - AL
A. L. Pappas - HO
B. K. Peele - BU
G. A. Porter - MV
G. A. Pullis - HO
D. W. Rice - HO
J. W. Schaefer - HO
P. L. Schmidt - WL
P. O. Schuh - IN

COVER SHEET ONLY TO

CORRESPONDENCE FILES:

THREE COPIES—PLUS ONE COPY FOR EACH ADDITIONAL FILING SUBJECT

Complete Memorandum to (Cont'd)

N. E. Snow - HO
S. G. Student, Jr. - HO
W. E. Thelen - MV
R. N. Watts - HO
T. J. West - MH
R. L. Whalin - HO
R. W. Wyndrum, Jr. - MH
R. M. Zeigler - HO
H. M. Zydney - HO

E-1932-D (2-62)

BELL TELEPHONE LABORATORIES
INCORPORATED

SUBJECT: Realization of FEN RC Input Networks
Case 38763-17

DATE: May 15, 1967

FROM: D. G. Marsh
MM 67-5316-3

MEMORANDUM FOR FILE

Introduction

In a building block technique of designing active RC filters, G. S. Moschytz (MM 67-5316-1, Active RC Filter Building Blocks Using Frequency Emphasizing Networks, Part I) has suggested breaking down complex filter networks into a small group of cascaded second order RC active networks. Identical building blocks are to be used for any desired filter configuration, and the building block consists of a second order RC approximation of the desired second order network cascaded with a frequency emphasizing network.

Various RC networks have been suggested by Moschytz for the particular type of filter desired, and in this memorandum, the realization of these RC networks for the low-pass, band-pass, high-pass, and resonator cases are discussed. Table 1 gives the general form of the transfer admittance, the RC network, and the parameters for each of these four cases. Curves have been obtained by use of the computer to simplify design of the filters.

For the frequency emphasizing network (FEN), there are two cases, medium selectivity FEN (MSFEN), and high selectivity FEN (HSFEN). The initial part of this discussion will be concerned with the MSFEN ($q_p < 10$),* while the HSFEN, which in general will be used only for band-pass filters, will be discussed later.

Medium Selectivity FEN

For the building block shown in Figure 1, assume that the desired transfer function has a denominator of the form $s^2 + \frac{\omega_n}{q_p} s + \omega_n^2$, that it is desired that the transmission zeros of the FEN cancel with the transmission poles of the RC network (which has a denominator of the form $s^2 + \frac{\omega_p}{q_R} s + \omega_p^2$), and that the poles of the FEN are to be the poles of the desired transfer function. Since the FEN transfer function is of the form

$$T_F(S) = \frac{s^2 + \frac{\omega_N}{q_N} s + \omega_N^2}{s^2 + \frac{\omega_N}{q_F} s + \omega_N^2}, \tag{1}$$

Moschytz shows that if

$$\frac{1}{RC} = \omega_N = \omega_p = \omega_n,$$

$$\frac{1}{2} \frac{\rho}{1+\rho} = q_N = q_R,$$

and

$$q_F = q_p,$$

* The terminology used here is the same as that used in the above mentioned memorandum by Moschytz.

then those above assumptions are obtained. Figure 1 shows that R, C, and ρ are the parameters of the twin-T of the **FEN**, and in the case of a symmetrical twin-T, $\rho = 1$. Moschytz suggests that the case of the symmetrical twin-T seems to be the best, and therefore, throughout this memorandum it will be assumed that $\rho = 1$ and thus that $q_N = q_R = \frac{1}{4}$.

A. Low-Pass Filter (LPF)

The known parameters for the LPF will, in general, be q_p, f_n, and the feedback resistor R_F, of the inverting operational amplifier of the **FEN**. One must then find the resistor R_Q of the **FEN** and the values of the five elements of the input RC network in order to obtain the desired LPF characteristics.

Given q_p and R_F, R_Q can readily be found from Figure 3, and the parallel combination of R_F and R_Q, which is an equivalent resistor K_A, is found from Figure 4.

Because space is at a premium and because thin film technology places definite limits on capacitance per substrate, it is desirable to use RC networks which have minimum capacitance. Use of equal capacitors will satisfy this constraint, and their value should be selected by the designer. With this equal capacitors constraint, the LPF equations of Table 1 have been solved and plotted in Figures 5 through 7.

Using Figure 5, for the given frequency, f_n, and the given capacitor, C, one obtains $\omega_n C$. Then for that $\omega_n C$, any sum of the three resistors, $R_1 + R_2 + R_3$, above the Realizability Limit (RL) will yield real positive values for R_1, R_2, and R_3. The circuit designer may choose any value of the sum above the RL for his $\omega_n C$, but for space conservation, it is suggested that the value not be much above that RL.

Using Figure 6, for the given $\omega_n C$, one obtains R_2. This value deviates by less than one percent per decade change in $R_1 + R_2 + R_3$, and has therefore been plotted as if it were independent of that sum.

Using Figure 7, for the given $\omega_n C$ and the selected $R_1 + R_2 + R_3$, one obtains R_1 (or R_3, because the network is symmetrical). Then the third resistor, R_3 (or R_1), is easily obtained. Unless the source impedance to the building block is high, the larger of R_1 or R_3 should be used at the input to the FEN to minimize the loading of the FEN operational amplifier, a loading effect which may be significant at high frequencies due to reduction by frequency compensation of the open loop gain.

It should be pointed out that the shaded areas of Figure 5 and Figure 7 are also realizable. However, in this region, R_2 begins to depend strongly on the sum, and, as seen by the dotted lines of Figure 7, so does R_1 and R_3.

Finally the net forward gain, g, at dc can be found by applying

$$g_{LPF}(0) = \frac{K_A}{R_1+R_2+R_3} .$$

An example will now be considered. Suppose one desires a LPF with a $q_p = 2$, and $R_F = 27k\Omega$, and an $f_n = 60kHz$.

Figure 3 gives $R_Q = 3.85k\Omega$.

Figure 4 gives $K_A = 3.35k\Omega$.

Let $C = 0.01\ \mu F$.

Figure 5 gives $\omega_n C = 38\times10^{-4}$ mhos, and

$(R_1+R_2+R_3)_{MIN} = 2.75k\Omega$. We select

$R_1 + R_2 + R_3 = 3.35k\Omega$(i.e., unity dc gain)

Figure 6 gives $R_2 = 150\Omega$.

Figure 7 gives $R_1 = 650\Omega$.

Then $R_3 = 3350 - 800 = 2550\Omega$.

The values obtained for this example are shown in the corresponding figures.

At lower frequencies, one might have to suffer some loss or use larger, discrete capacitors.

B. Band-Pass and High-Pass Filters

Examination of Table 1 shows that the band-pass and high-pass filters (BPF and HPF) have one component less than the LPF, and, therefore, are much more easily analyzed than the LPF. Two cases have been considered for both the BPF and the HPF: one case with equal capacitors (Figure 8), and one case with equal time constants for the two RC products (Figure 9).

The given parameters for both the BPF and the HPF are the same as for the LPF (q_p, R_F, and f_n) and therefore R_Q and K_A are found from Figures 3 and 4, as before.

In the case of equal capacitors, we let one resistor be R and the other be ρR. Then solving the four sets of equations (two sets each, "a" and "b," for the BPF and the HPF) of Table 1 with $q_R = \frac{1}{4}$, one finds that $\rho = 0.343$ for the "a" configurations of the Table and that, due to reciprocity, the "a" and "b" configurations for each filter are actually identical. For a given frequency and a given capacitor, Figure 8 gives the resistor, R. The resultant BPF and HPF networks, with the second resistor, 0.343R, are also shown in the Figure. The forward gains are

$$g_{BPF}(\omega_n) = R_F \cdot K_R \frac{q_R}{\omega_n} = R_F \cdot \frac{1}{\rho r} \cdot \frac{1}{4} \cdot \sqrt{\rho}$$

$$= \frac{1}{4\sqrt{\rho}} \cdot \frac{R_F}{R} = 0.426 \frac{R_F}{R} ,$$

and $$g_{HPF}(\infty) = K_A K_R = K_A \cdot \frac{1}{\rho R} = 2.92 \frac{K_A}{R},$$

where

$$\omega_n = \frac{1}{\sqrt{\rho}\ RC} = \frac{1}{0.586RC} .$$

In the case of equal time constants, the resistors and capacitors of Table 1 are designated R, C, ρR, and C/ρ. Again the four sets of equations are solved for $q_R = \frac{1}{4}$, and, for the "a" configuration, $\rho = \frac{1}{2}$. Again, due to reciprocity, the two configurations for each filter are identical. For a given frequency and capacitor, Figure 9 gives the resultant value of R, and the two networks are also given. The forward gains for this case are

$$g_{BPF}(\omega_n) = R_F \cdot K_R \frac{q_R}{\omega_n} = R_F \cdot \frac{2}{R} \cdot \frac{1}{4} = \frac{R_F}{2R}$$

and $$g_{HPF}(\infty) = K_A \cdot K_R = \frac{2K_A}{R} .$$

where

$$\omega_n = \frac{1}{RC} .$$

C. Resonator

Two cases of the resonator are considered. The combination of the two resulting cases covers the frequency range of

$$0 < f_Z < 3.7f_n.$$

The given parameters for the resonator are q_p, R_F, f_n, and f_z. R_Q and K_A are found as before from Figures 3 and 4.

For case I, the capacitors are set equal, $R_1 = \rho R$, $R_2 = R$, and $R_3 = \alpha R$ and thus $\omega_Z = \frac{1}{RC}$ and $\omega_n = \dfrac{1}{\sqrt{\frac{\alpha\rho}{1+\alpha+\rho}}\,RC}$.

Solving the equations of Table 1 for $q_R = \frac{1}{4}$, one finds that the case I circuit can be realized if $0 < f_Z < 0.27f_n$. Given f_Z/f_n, the dashed curve of Figure 10 gives ρ, and for that value of ρ, the solid curve gives the corresponding α. Since $2\pi f_Z = \frac{1}{RC}$, the BPF and HPF equal time constant curves (Figure 9) can be used to find R for a given capacitor by using f_Z in place of f_n.

For case II, $R_1 = R$, $R_2 = \rho R$, $R_3 = \alpha R$, $C_2 = C/\rho$, and $C_1 = C(1+\rho+\alpha)$ where now $\omega_Z = \frac{1}{RC}$ and $\omega_n = \frac{1}{\sqrt{\alpha}\ RC}$. Solving the equations of Table 1 for $q_R = \frac{1}{4}$, one finds that the case II network can be realized for the range

$$0.27f_n < f_Z < 3.7f_n.$$

The ratio f_Z/f_n is used in Figure 11 to find α and ρ, and R and C are again found from the BPF and HPF equal time constant curves by using f_Z in place of f_n.

In both cases, the dc gain is

$$g_{RES}(0) = \left(\frac{1}{1+\alpha+\rho}\right)\frac{K_A}{R}.$$

<u>High Selectivity FEN (HSFEN)</u>

The band-pass filter is the only case for which the HSFEN ($q_p > 10$) given in Figure 2 is important. RC pole - FEN zero cancellation is not possible in this HSFEN because

$$q_N = \frac{1}{2}\left(\frac{1+r}{r}\right) > \frac{1}{2},$$

where

$$r = (1+\rho)\frac{R}{R_L} = \frac{2R}{R_L}$$

for $\rho = 1$. Because that cancellation is not obtainable, Moschytz suggests a MSFEN with $q_p = \frac{1}{2}\left(\frac{1+r}{r}\right)$ and $q_N = \frac{1}{4}$ be placed in cascade with the RC-HSFEN combination.

Considering just the HSFEN, one finds that it has

$$q_p = \frac{1+r+R_F/R_Q}{2r} .$$

Figure 12 yields R_F/R_Q for given q_p and r, and then K_A is readily determined from

$$K_A = R_F\left(\frac{1}{R_F/R_Q+1}\right) .$$

The RC network and the gain can be found just as in the MSFEN case. It is important to note that any passband loss due to $g(\omega_n) < 1$ can be made up by cascading the MSFEN and the HSFEN through a resistor R_1 and setting

$$R_1 = g(\omega_n) \cdot R_{FM}$$

where R_{FM} is the MSFEN feedback resistor.

Restrictions

The present design does have several definite frequency restrictions. Above 150kHz, presently available operational amplifiers are sufficiently reduced in loop gain that they cannot be assumed to be stable, "high gain" devices.

A BPF can be designed for unity gain at passband down to a few hundred hertz (depending on the value of R_F) for MSFEN and even lower for HSFEN if a MSFEN is cascaded with it. However, for LPF and HPF the low frequencies exhibit a "q-gain" limitation. For example, if q_p is relatively large, then $K_A = \frac{R_F}{4q_p-1}$ might be quite small. Since $g_{LPF}(0)$ and $g_{BPF}(\infty)$ are directly proportional to the ratio of K_A to the resistors of the RC network, those resistors must also be small in order not to have much (if any) loss. However, this requires large capacitors, and thin film technology might not allow the required valves. A second generation of RC active filters is presently being designed which will extend the LPF and HPF filter ranges down to a few hundred hertz.

Realization of the resonator in the vicinity of $0.27f_n = f_Z$ and $3.7f_n = f_Z$ might require excessively large or small resistors as one approaches those values. A similar restriction occurs if one attempts to build a resonator with $f_Z \ll 0.27f_n$.

Conclusions

A set of curves have been presented which will allow the designer to construct a variety of RC active filters. These active, two pole devices can be cascaded together to form almost any transmission characteristics which could be obtained with conventional RLC circuits. The major frequency limitation of the present design exists because of the limits on capacitor size imposed by thin film technology.

D. G. Marsh

HO-5316-DGM-CAT

D. G. MARSH

Att.
Tables (1)
Figures (1 - 12)

TABLE I: TRANSFER ADMITTANCE OF SOME 2ND ORDER RC TWO PORTS

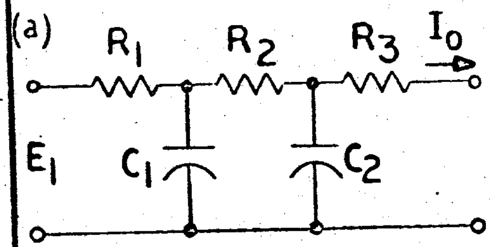
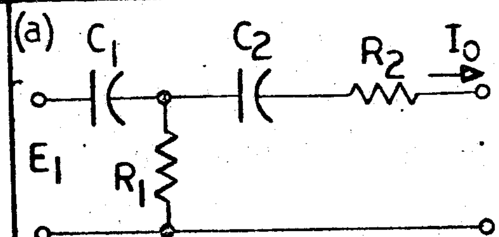
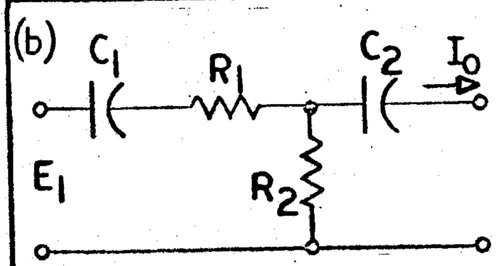
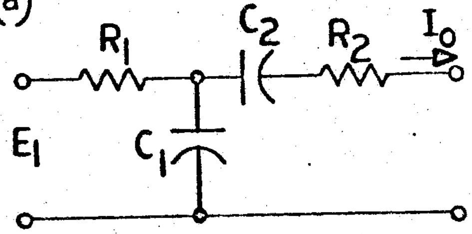
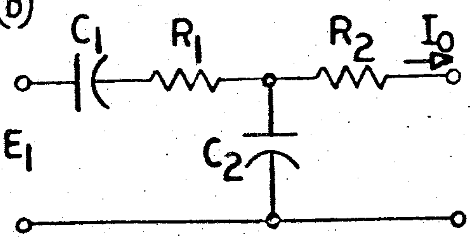
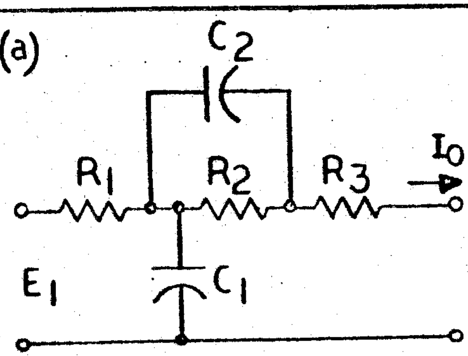
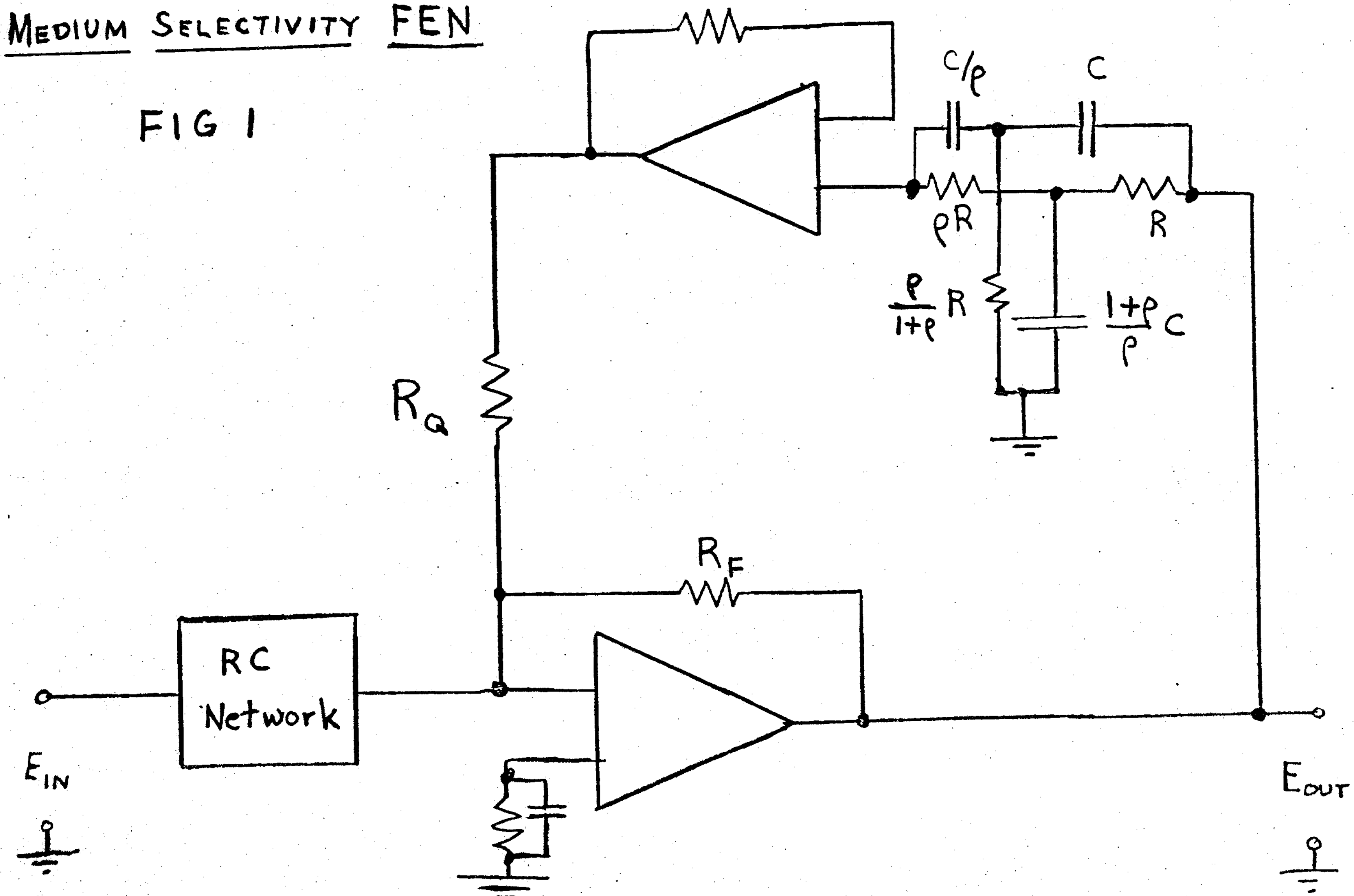
TRANSFER ADMITTANCE	RC NETWORK	PARAMETER-COMPONENT RELATIONS
1) LOW-PASS FILTER $K_R \cdot \dfrac{1}{S^2 + \frac{\omega_p}{q_R} S + \omega_p^2}$	(a) 	$K_R = \dfrac{1}{R_1 R_2 R_3 C_1 C_2}$; $\omega_p^2 = \dfrac{R_1 + R_2 + R_3}{R_1 R_2 R_3 C_1 C_2}$ $q_R = \dfrac{\sqrt{R_1 R_2 R_3 (R_1 + R_2 + R_3) C_1 C_2}}{R_1 (R_2 + R_3) C_1 + R_3 (R_1 + R_2) C_2}$
2) HIGH-PASS FILTER	(a) 	$K_R = \dfrac{1}{R_2}$; $\omega_p^2 = \dfrac{1}{R_1 R_2 C_1 C_2}$; $q_R = \dfrac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2) + R_2 C_2}$
$K_R \cdot \dfrac{S^2}{S^2 + \frac{\omega_p}{q_R} S + \omega_p^2}$	(b) 	$K_R = \dfrac{1}{R_1}$; $\omega_p^2 = \dfrac{1}{R_1 R_2 C_1 C_2}$; $q_R = \dfrac{\sqrt{R_1 R_2 C_1 C_2}}{(R_1 + R_2) C_1 + R_2 C_2}$

TABLE I CONT'D

TRANSFER ADMITTANCE	RC NETWORK	PARAMETER - COMPONENT RELATIONS
3) BAND-PASS FILTER	(a) 	$K_R = \frac{1}{R_1 R_2 C_1}$; $\omega_p^2 = \frac{1}{R_1 R_2 C_1 C_2}$; $q_R = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1(C_1 + C_2) + R_2 C_2}$
$K_R \cdot \frac{S}{S^2 + \frac{\omega_p}{q_R} S + \omega_p^2}$	(b) 	$K_R = \frac{1}{R_1 R_2 C_2}$; $\omega_p^2 = \frac{1}{R_1 R_2 C_1 C_2}$; $q_R = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(R_1 + R_2) C_1 + R_2 C_2}$
4) RESONATOR $K_R \cdot \frac{S + \omega_Z}{S^2 + \frac{\omega_P}{q_R} S + \omega_p^2}$	(a) 	$K_R = \frac{1}{R_1 R_3 C_1}$; $\omega_Z = \frac{1}{R_2 C_2}$; $\omega_p^2 = \frac{R_1 + R_2 + R_3}{R_1 R_2 R_3} \cdot \frac{1}{C_1 C_2}$; $q_R = \frac{\sqrt{R_1 R_2 R_3 (R_1 + R_2 + R_3) C_1 C_2}}{(R_2 + R_3) R_1 C_1 + (R_1 + R_3) R_2 C_2}$

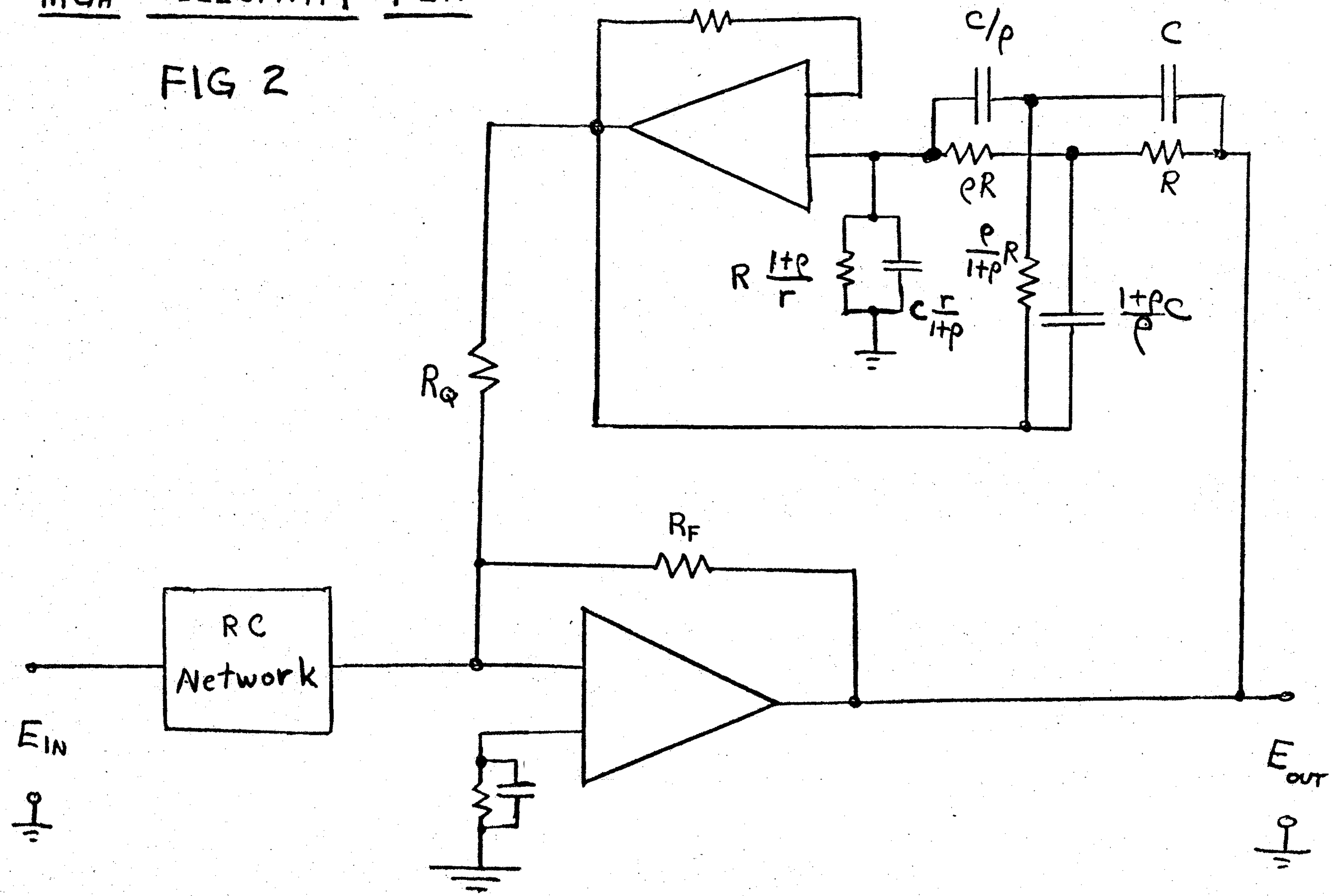
Medium Selectivity FEN

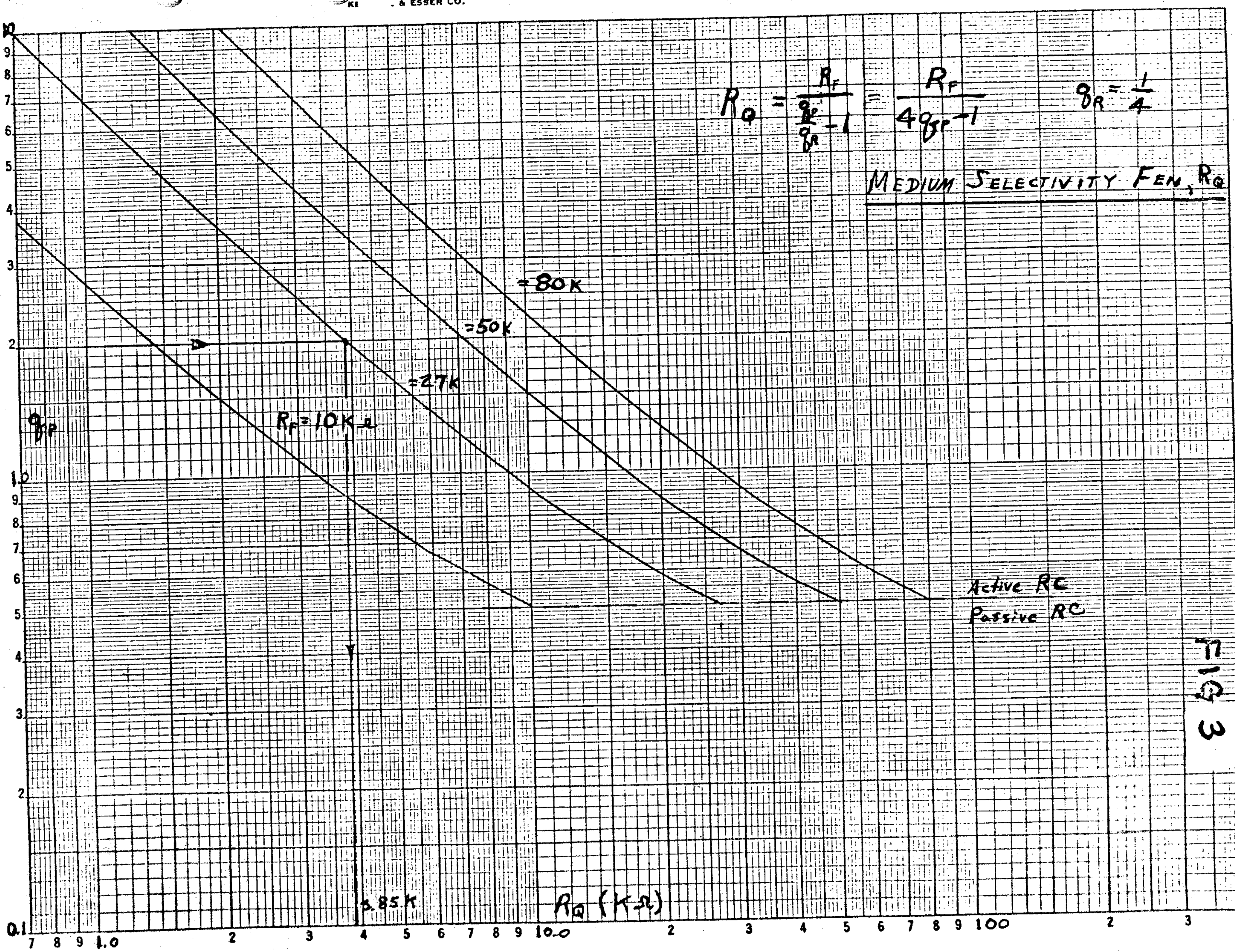
FIG 1



HIGH SELECTIVITY FEN

FIG 2





$R_Q = \frac{R_F}{\frac{q_P}{q_R} - 1} = \frac{R_F}{4q_P - 1}$ $q_R = \frac{1}{4}$
MEDIUM SELECTIVITY FCN, R_Q
=80K
=50K
=27K
R_F=10K Ω
q_P
Active RC
Passive RC
3.85K
R_Q (KΩ)

FIG 3

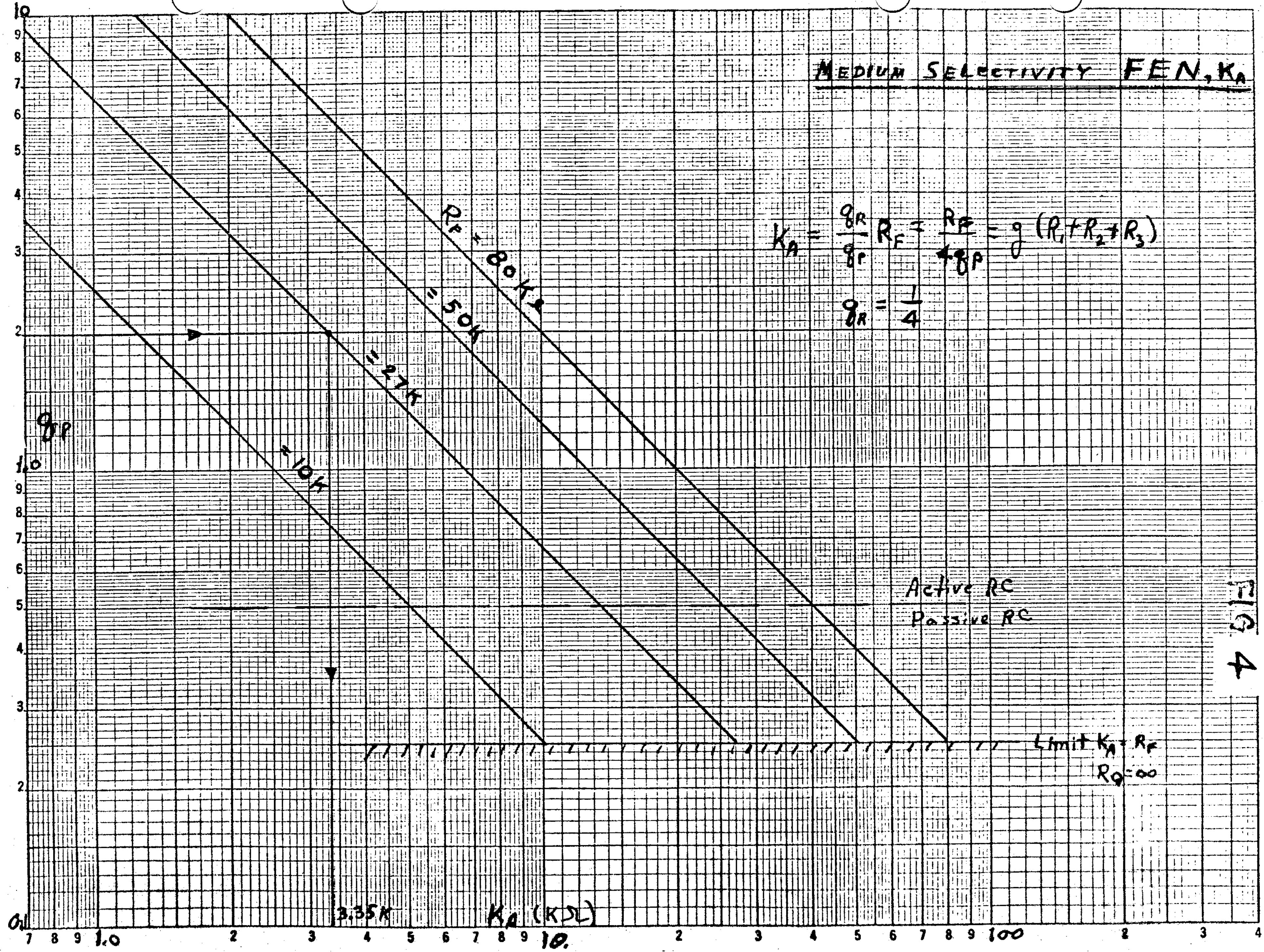
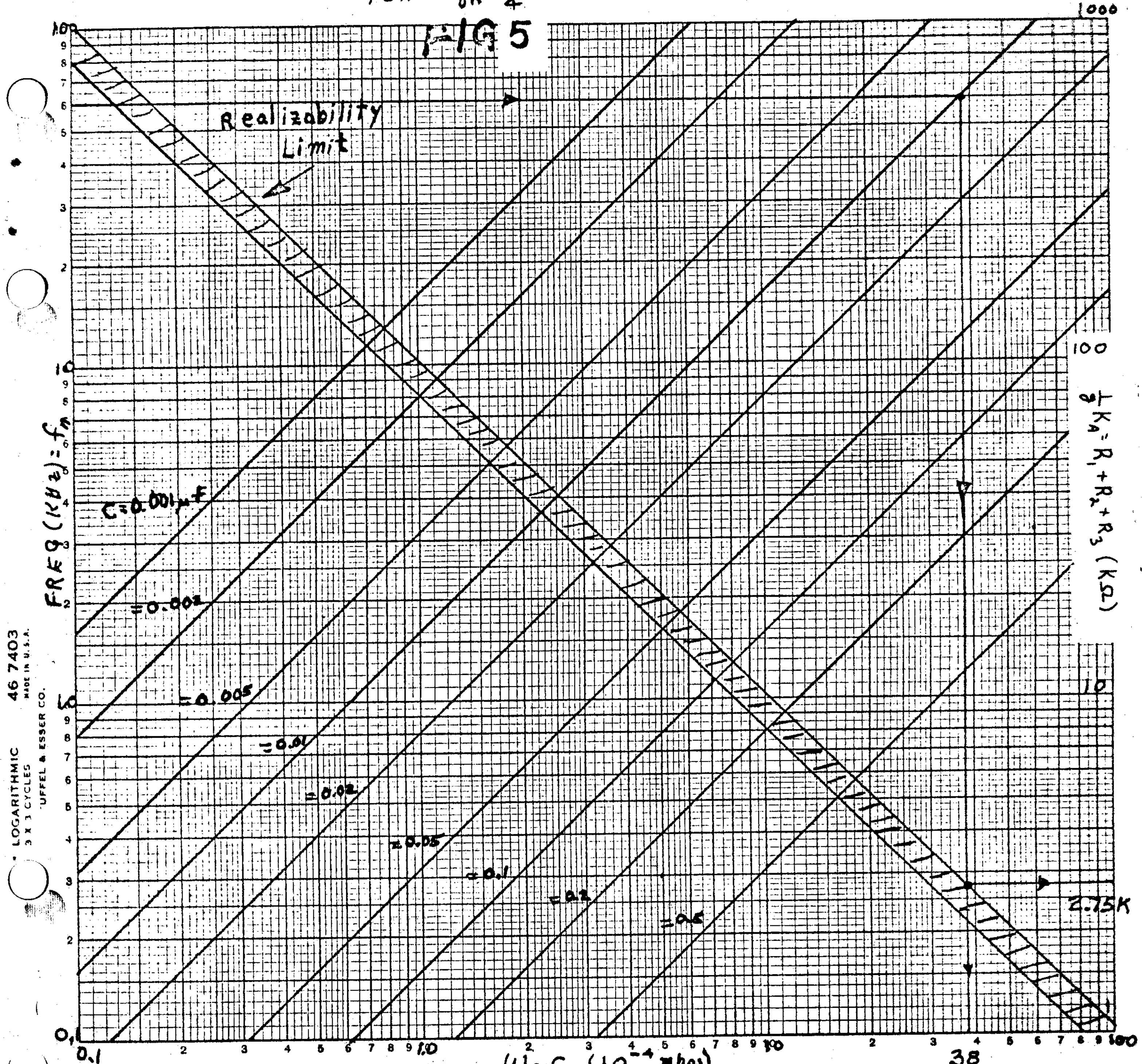

MEDIUM SELECTIVITY FEN, K_A
$K_A = \frac{g_R}{g_F} R_F = \frac{R_F}{4 g_P} = g(R_1 + R_2 + R_3)$
$g_R = \frac{1}{4}$
R_F = 80K
= 50K
= 27K
= 10K
g_P
Active RC
Passive RC
Limit $K_A = R_F$
$R_Q = \infty$
3.35K
K_A (KΩ)

FIG 4

LOW PASS FILTER
REALIZABILITY LIMITATIONS FOR $g_A = \frac{1}{4}$

FIG 5

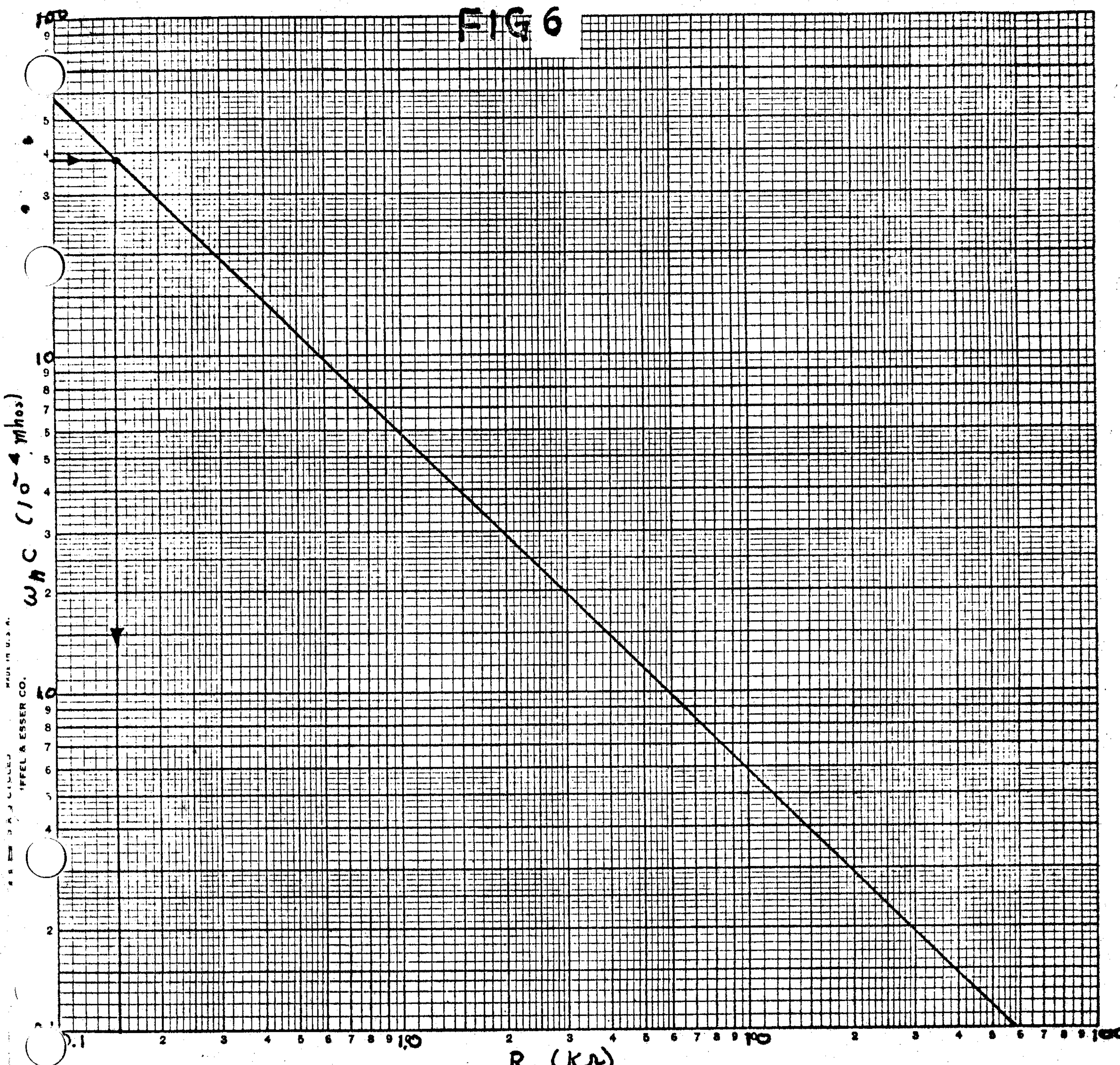


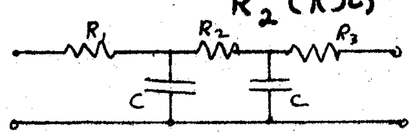
The freq and capacitor determine $\omega_n C$. For that $\omega_n C$, any $R_1 + R_2 + R_3$ above the "Realizability Limit" may be used.

LOW PASS FILTER

CENTER RESISTOR

FIG 6

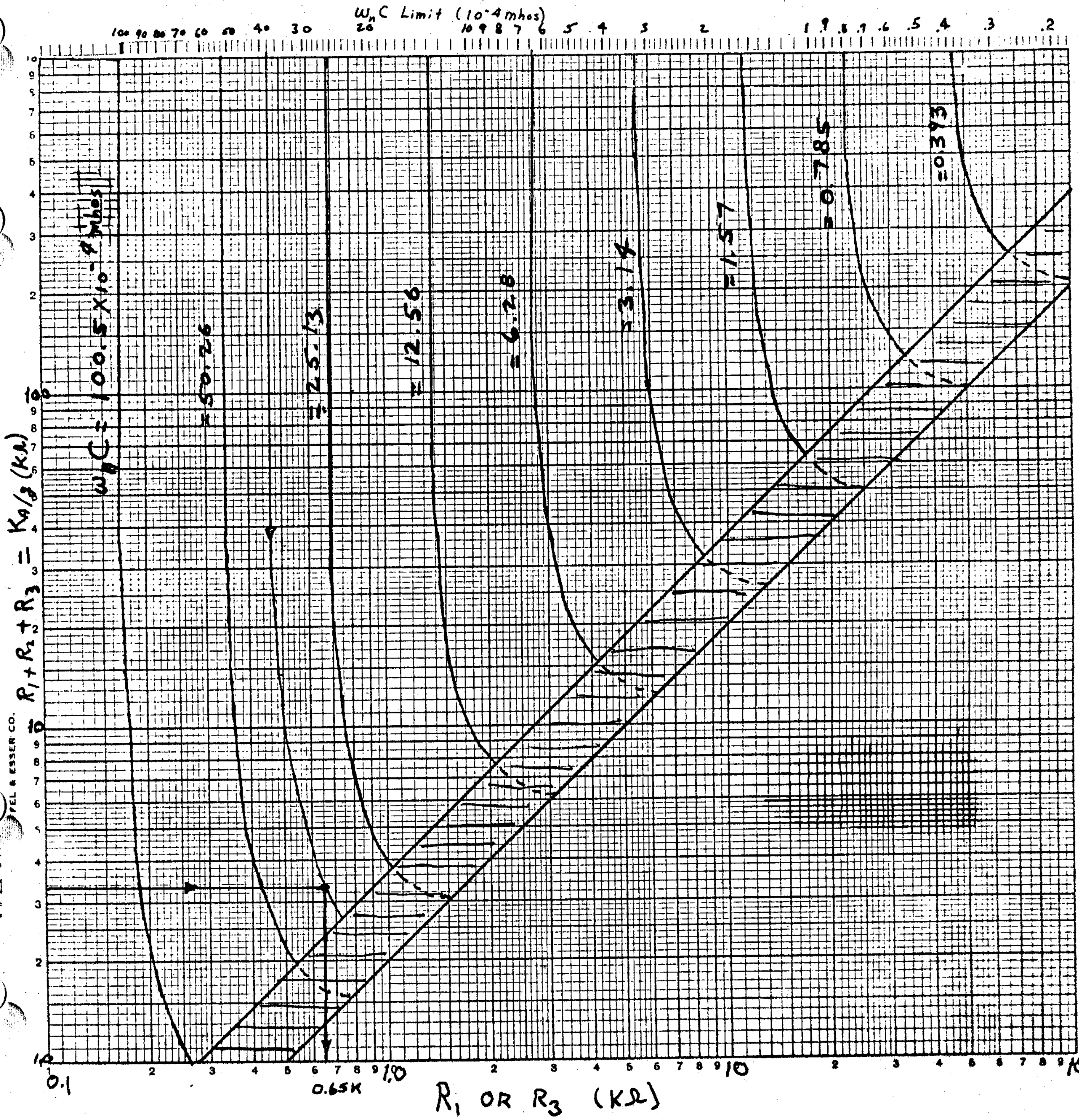


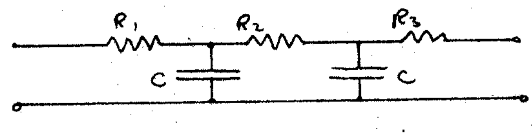


Low Pass Filter

Terminal Resistors

FIG 7

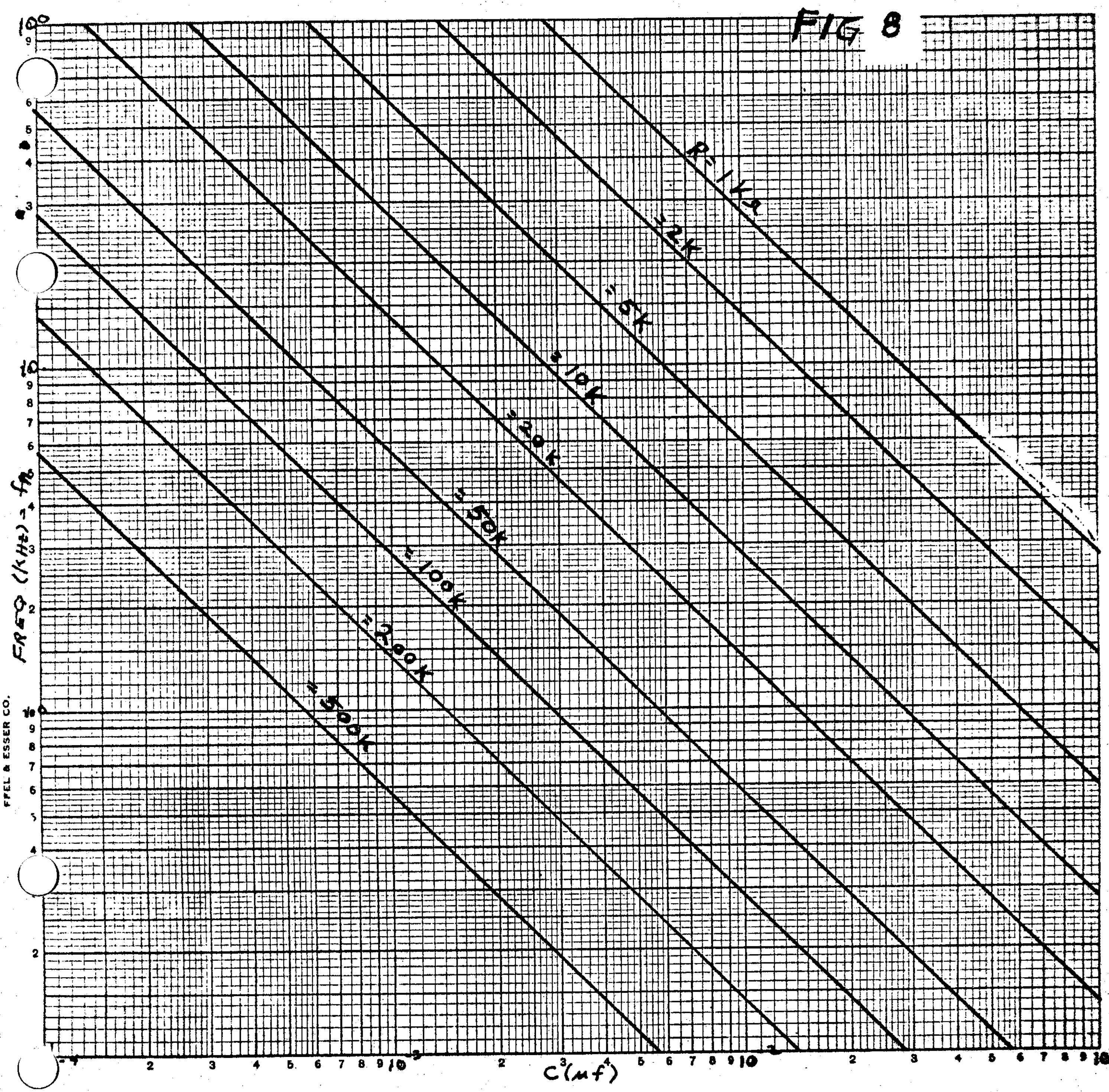




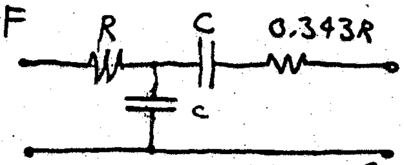
BAND-PASS AND HIGH-PASS FILTERS

IDENTICAL CAPACITORS

FIG 8



BPF

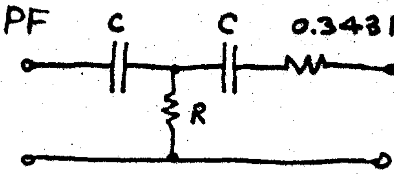


$g_{BPF}(\omega_R) = 0.426 \frac{R_F}{R}$

$$2\pi f_R = \frac{1}{0.586\,RC}$$

$$q_R = \frac{1}{4}$$

HPF

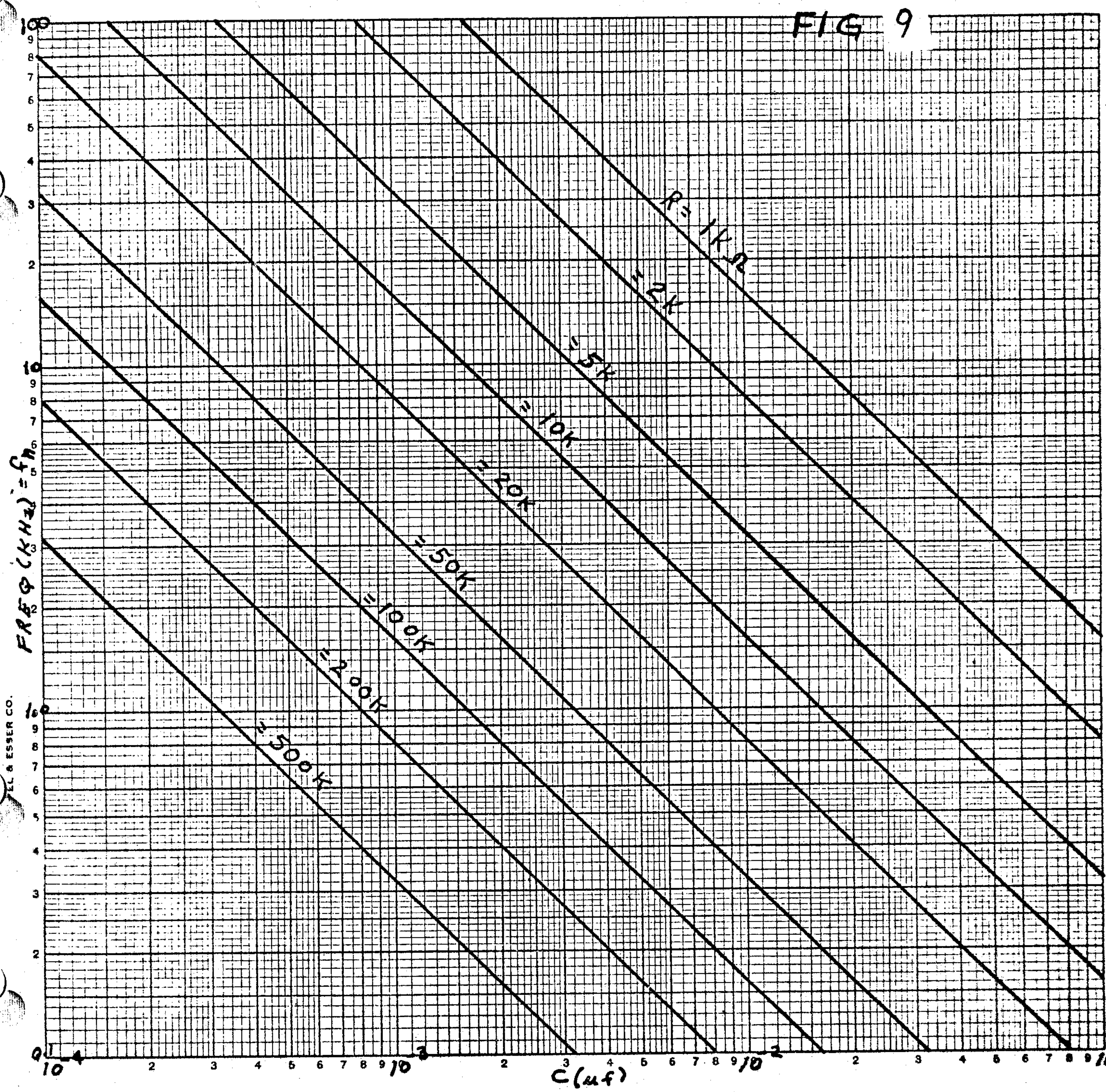


$g_{HPF}(\infty) = 2.92 \frac{K_A}{R}$

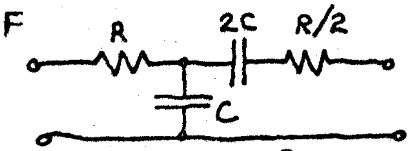
BAND-PASS AND HIGH-PASS FILTERS

IDENTICAL TIME CONSTANTS

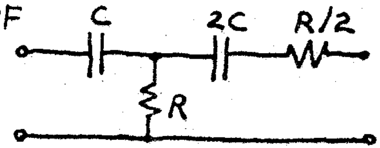
FIG 9



BPF



HPF



$$2\pi f_n = \frac{1}{RC}$$

$$g_{BPF}(\omega_n) = \frac{R_F}{2R} \qquad g_R = \frac{1}{4} \qquad g_{HPF}(\infty) = \frac{2K_A}{R}$$

RESONATOR I

FIG 10

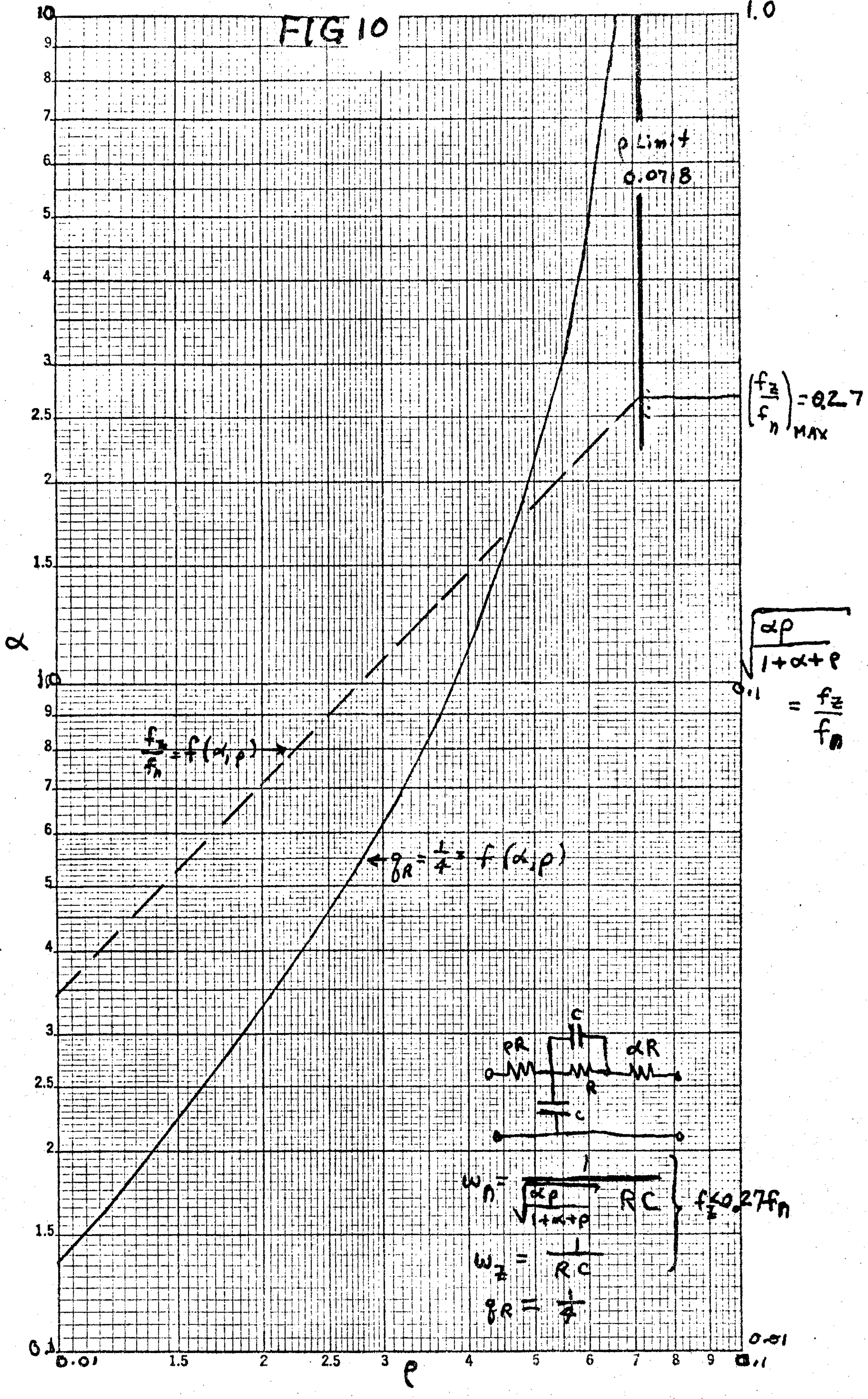
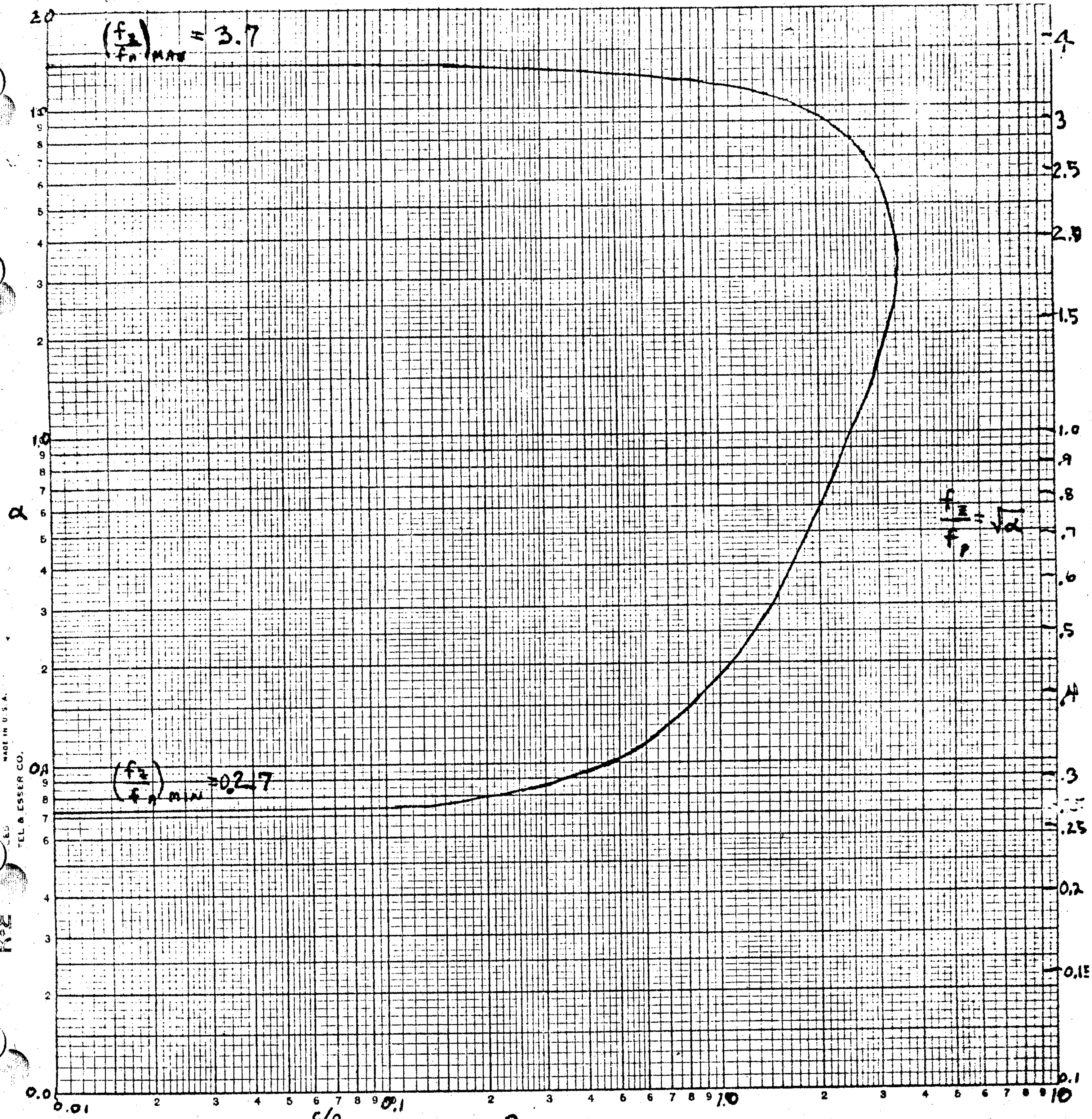
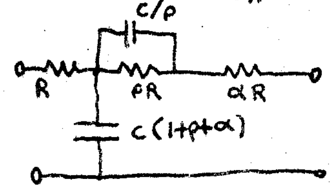


FIG 11
RESONATOR II





$$\left.\begin{aligned} \omega_z &= \frac{1}{RC} \\ \omega_p &= \frac{1}{\sqrt{\alpha}\,RC} \end{aligned}\right\} \quad 0.27\, f_p < f_z < 3.7\, f_p$$

$$\delta_R = \frac{1}{4}$$

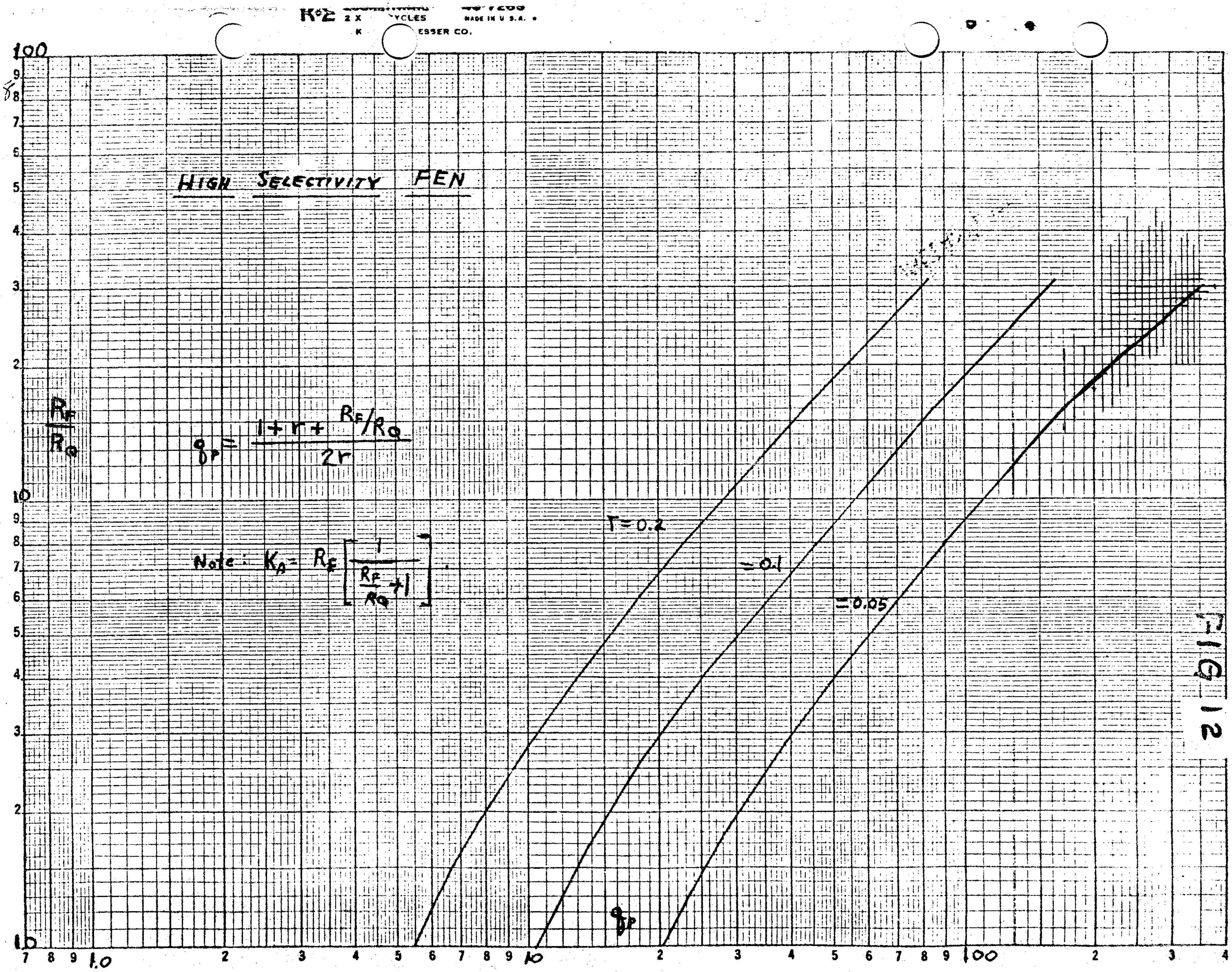

HIGH SELECTIVITY FEN
$q_P = \frac{1 + r + R_F/R_Q}{2r}$
Note: $K_A = R_F \left[\frac{1}{\frac{R_F}{R_Q} + 1} \right]$
R_F/R_Q
q_P
r = 0.2
= 0.1
= 0.05
K+E 2 X CYCLES
MADE IN U.S.A.
ESSER CO.

FIG 12