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TITLE— Active RC-Tantalum Integrated Circuit Filter Design for High Performance Specifications - Part I.

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ABSTRACT

It is generally accepted that active RC filters with moderately sharp frequency selective characteristics can be readily implemented as Tantalum Integrated Circuits. This class of filters is roughly known to include those whose dominant poles have $Q < 50$, at suitably low frequencies and impedance levels.

This study has determined the feasibility of providing high performance filter network characteristics (such as the N3 channel band filter) with active lumped RC-Tantalum Integrated Circuits. Minimum multiparameter sensitivity techniques provide a theoretical design procedure which most efficiently exploits the tantalum technology.

A ten-stage active RC filter (four passive stages, six active stages, and isolation amplifiers between them) is proposed to take maximum advantage of the tantalum I.C. technology.

A direct substitution of the present N3 crystal filter, at carrier frequencies above 140 KHz, requires ±0.01% initial RC product adjustment and an operating temperature range of less than 10°C to meet and maintain the channel filter specifications. Capacitor aging and humidity stability of .01% is necessary in critical circuits to preclude field adjustment. Presently, this cannot be met by tantalum thin film capacitors, but may be achieved using appliqued capacitors in four critical subcircuits.

An alternate technique for meeting channel band requirements utilizing current hybrid Tantalum Integrated Circuit technology, without appliqued components may be provided by "pregrouping"; i.e., modulating the voice channel to 20-36 KHz, removing one sideband with the active RC channel band filter, then modulating to the N3 line frequencies.

Computer simulation indicates the proposed 140 KHz channel band filter meets design objectives with some minor modifications; it has been carefully analyzed to determine the deviation of its gain characteristics from all anticipated device and component tolerances.

Part II will discuss the experimental results of a 140 KHz breadboard realization of the filter, as well as more precise requirements on the operational amplifiers used.

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FROM: S. C. Lee
R. W. Wyndrum, Jr.

MM 67-2623-2

MEMORANDUM FOR FILE

I. INTRODUCTION

Sipress [1] and Witt [2] demonstrated that the difficulty of making practical lumped active RC filters meeting prescribed high selectivity characteristics is in maintaining the characteristics despite anticipated component variations. This is the sensitivity problem. Whether active RC filters may be made competitive with crystal filters in economy and performance depends on how insensitive the filter characteristics can be made to component variations. In 1959 Witt showed that an active RC channel band filter was technically, but not economically, feasible. "Technically feasible", implied that by carefully preselecting elements with initial values within the required limits, one can build an active RC network which meets the nominal channel band specification.

A significant difference between a conventional active RC network and a Tantalum Integrated Circuit is that the former must be made of selected components whereas the latter has a better means of meeting requirements. In this memorandum, a new technique, "functional adjustment", is introduced to accommodate the initial tolerance problem in making thin film circuits; the network function is closely achieved by adjusting several selected resistors, rather than the individual trimming of every element to its nominal value.

It is possible to adjust tantalum film resistors by anodization to within .01% and this can often be made to compensate for capacitor imprecision. At the present time a tantalum film resistor may be trimmed to greater precision than the capacitor and thus it eventually limits the initial trimmed network function tolerance. In addition to the initial trimmed tolerance, aging and temperature tolerances must be considered during the circuit design. The gain deviations due to initial, temperature, and aging tolerances are discussed in detail in this memorandum, giving results based on a computer simulated worst-case deviation study.

II. REALIZATION TECHNIQUES

A. Approximation

The first step in network synthesis is the approximation of the prescribed network characteristic by a realizable mathematical function, either in the frequency domain or the time domain. In this memorandum, frequency domain approximation is considered [3].

Consider the bandpass characteristic (Fig. 1(a)) whose pole-zero representation is shown in Fig. 1(b). The 3 dB bandwidth, B.W. = $2\sigma_c$, where σ_c is the magnitude of the real part of the complex poles. The Q of the poles is

$$Q = \frac{\omega_c}{2\sigma_c} = \frac{\text{center frequency}}{\text{bandwidth}} . \tag{1}$$

This frequency response can be realized by an LC tuning circuit (Fig. 1(c)), as well as an inductorless active RC network (Fig. 1(d)). A pair of complex poles at $\pm j\omega_c$ (with real part $\sigma_c \ll \omega_c$) results in a "peak" in the frequency response at ω_c. A pair of imaginary zeros at $\pm j\omega_o$ corresponds to zero transmission at ω_o.

With these building blocks, one can construct the pole-zero configuration which corresponds to a bandpass frequency response, shown in three dimensions in Fig. 2. The horizontal plane is the s-plane with a number of high Q complex poles distributed near the passband. If we put one or more imaginary zeros just outside the passband, and at the origin, an excellent bandpass characteristic results. With the vertical axis representing the magnitude of the voltage gain, the frequency response corresponding to the pole-zero configuration is shown in Fig. 2.

B. Decomposition and Cascade Synthesis

Suppose the bandpass characteristic (Fig. 3(a)) can be approximated by the pole-zero representation of Fig. 3(b):

$$G(s) = \frac{s\left(s^2 + \omega_{o1}^2\right)\left(s^2 + \omega_3^2\right)}{\left[s^2 + 2\sigma_1 s + \left(\sigma_1^2 + \omega_1^2\right)\right]\left[s^2 + 2\sigma_2 s + \left(\sigma_2^2 + \omega_2^2\right)\right]\left[s^2 + 2\sigma_3 s + \left(\sigma_3^2 + \omega_3^2\right)\right]} . \qquad (2)$$

Note that we are allowed to add any number of poles and zeros in the LHP to a network function so long as the poles cancel the zeros:

$$G(s) = \frac{s\left(s^2 + \omega_{o1}^2\right)\left(s^2 + \omega_{o2}^2\right)}{\left[s^2 + 2\sigma_1 s + \left(\sigma_1^2 + \omega_1^2\right)\right] \cdots \left[s^2 + 2\sigma_3 s + \left(\sigma_3^2 + \omega_3^2\right)\right]} \frac{(s+a)(s+b)}{(s+a)(s+b)} \qquad (3)$$

This technique will be employed later to achieve a convenient mathematical decomposition. It is undesirable to realize a voltage transfer function having a large number of poles and zeros by a single network: we have no well-defined control on the number of circuit elements, or on the tolerance of the network frequency response $|G(j\omega)|$ due to circuit element tolerances. Therefore, we decompose $G(s)$ into a product of several simpler functions, realize each function by a separate network, and optimize the individual networks. For instance, (3) may be decomposed into n rational functions, for which the orders of the numerator and denominator polynomials usually do not exceed two, namely:

$$G(s) = \frac{\left(s^2 + \omega_{o1}^2\right)}{(s+a)(s+b)} \cdots \frac{(s)}{s^2 + 2\sigma_3 s + \left(\sigma_3^2 + \omega_3^2\right)} = G_1(s) \cdots G_n(s) \qquad (4)$$

This mathematical decomposition of G(s) permits each rational function $G_i(s)$ to be realized by a simple network. This leads to the cascade synthesis with isolation amplifiers between stages (Fig. 3(c)). $G_1(s)$ and $G_n(s)$ of Eq. (4) can be realized as a twin-tee network and an active RC network, respectively.

C. Sensitivity and Tolerance

For a given G(s), many different circuit realizations exist. For a specific realization, G(s) becomes a function of the circuit elements, some or all of which may deviate from the nominal design values in the physical network. For example, one term in (4) can be realized by the active RC network of Fig. 1(d), and can be expressed in terms of network parameters:

$$G(s) = \frac{V_{out}}{V_{in}} = \frac{-KR_2C_2s}{(1+K)R_1R_2C_1C_2s^2 + [R_1(C_1+C_2) + R_2C_2]\ s + 1} \qquad (5)$$

If only one network parameter e_i is allowed to vary from nominal and the rest of the network parameters are fixed at their design values, G(s) is a <u>single parameter network function</u>, denoted by $G(e_i, s)$. If there are n network parameters which may vary from nominal, then G(s) is a <u>multiparameter network function</u>, denoted by $G(e_1, e_2, \ldots, e_n, s)$.

<u>Single Parameter Sensitivity</u>

The single parameter sensitivity [4,5] of a network function $G(e_i, s)$ is defined as

$$S_{e_i}^{G} \equiv \frac{d(\ln G)}{d(\ln e_i)} \tag{6}$$

$S_{e_i}^{G}(j\omega)$ can be expressed in terms of the attenuation sensitivity (its real part) and the phase sensitivity (its imaginary part).

$$S_{e_i}^{G}(j\omega) = \mathrm{Re}\left[S_{e_i}^{G}(j\omega)\right] + j\ \mathrm{Im}\left[S_{e_i}^{G}(j\omega)\right] \tag{7}$$

$$= \frac{d\alpha(\omega)}{de_i} e_i + j\, \frac{d\beta(\omega)}{de_i} e_i \tag{8}$$

where $\alpha(\omega) = \ln |G(j\omega)|$ (the attenuator), and $\beta(\omega) = \mathrm{Arg}\ G(j\omega)$ (the phase). In the design of high performance filters, our attention is focused on the study of attenuation sensitivity. For small variations of e_i, the attenuation sensitivity

$$R_e\left[S_{e_i}^{G}(j\omega)\right] = \frac{d\alpha(\omega)}{de_i} e_i \triangleq S_{e_i}^{|G(j\omega)|} \cong \frac{\Delta|G(j\omega)|/|G(j\omega)|}{\Delta e_i/e_i} \tag{9}$$

indicates the percentage change of gain, resulting from a 1% change in e_i. If a given element tolerance ε_i is assumed to be small, then the gain deviation is

$$\frac{\Delta|G(j\omega)|}{|G(j\omega)|} \cong \pm\ \varepsilon_i \cdot S_{e_i}^{|G(j\omega)|}\ ,\ \text{where}\ \varepsilon_i = \frac{\Delta e_i}{e_i}\ . \tag{10}$$

Equation (10) relates the tolerance $\pm\varepsilon_i$ to the gain deviation of $|G(j\omega)|$.

Multiparameter Sensitivity

As an extension of single parameter sensitivity, multiparameter sensitivity may be derived from Eqs. (6)-(10); see [6]-[9]. Here we shall be interested in the gain deviation due to all the parameters. Assume that $\pm\varepsilon_1$, $\pm\varepsilon_2$, ..., $\pm\varepsilon_n$ are the tolerances of e_1, e_2, ..., e_n, respectively, and are assumed to be small. Using a first order approximation,

$$\begin{aligned}\frac{\Delta|G(j\omega)|}{|G(j\omega)|} &= \pm\varepsilon_i\ S_{e_i}^{|G(j\omega)|}\ \pm\varepsilon_2\ S_{e_2}^{|G(j\omega)|}\ \pm \dots \pm\varepsilon_n \cdot S_{e_n}^{|G(j\omega)|} \\ &= \sum_{i=1}^{n} \pm e_i\ S_{e_i}^{|G(j\omega)|}\end{aligned} \tag{11}$$

From Eq. (11) the total gain deviation is the algebraic sum of the gain deviations due to each parameter. Large parameter tolerances and small sensitivities may thus have the same final effect as small parameter tolerances and large sensitivities. This multiparameter deviation minimization approach will be applied to the circuit design of the subject filters.

From (11), the worst-case gain-deviation (WCGD) is

$$\mathrm{WCGD} \triangleq |\pm\varepsilon_1|\cdot|S_{e_1}^{|G(j\omega)|}|+|\varepsilon_2|\cdot|S_{e_2}^{G(j\omega)}| + \ldots + |\pm\varepsilon_n|\cdot|S_{e_n}^{|G(j\omega)|}| \tag{12}$$

For any passive RC realization [10], or active RC realization whose active element tolerance is negligibly small (see Section IV),

$$S_R^G = S_C^G = \frac{d(\ln G)}{d(\ln s)} \tag{13}$$

where

$$S_R^G = \sum_{i=1}^{r} S_{R_i}^G, \quad S_C^G = \sum_{i=1}^{c} S_{C_i}^G,$$

and r and c are the number of resistors and capacitors in the network. From (13),

$$\sum_{i=1}^{r} \mathrm{Re}\left[S_{R_i}^G(j\omega)\right] = \sum_{i=1}^{c} \mathrm{Re}\left[S_{C_i}^G(j\omega)\right] = \mathrm{Re}\left[\frac{d(\ln G(j\omega))}{d(\ln(j\omega))}\right] \tag{14(a)}$$

or

$$\sum_{i=1}^{r} S_{R_i}^{|G(j\omega)|} = \sum_{i=1}^{c} S_{C_i}^{|G(j\omega)|} = \mathrm{Re}\left[\frac{d(\ln(G(j\omega))}{d(\ln(j\omega))}\right] \tag{14(b)}$$

It is clear, in general, for two network realizations of $G_i(s)$, say $G_1(s)$ and $G_2(s)$, that

$$G_1(s) = G_2(s) \tag{15(a)}$$

and for $m > n$

$$\therefore \sum_{1}^{n} S_{R_i}^{G_1}(s) = \sum_{1}^{m} S_{R_j}^{G_2}(s) \tag{15(b)}$$

$$\sum_{1}^{n} |S_{R_i}^{G_1}(s)| \leq \sum_{1}^{m} S_{R_j}^{G_2}(s) \tag{15(c)}$$

or for equal ε,

$$WCGD_1 \leq WCGD_2 \tag{15(d)}$$

Consider two realizations: one contains r_1 resistors and c_1 capacitors, the other contains r_2 resistors and c_2 capacitors. Assume that $r_2 > r_1$ and $c_2 > c_1$, and the individual tolerances of the resistors and capacitors of the two networks are the same. <u>It is evident from (12) and (15) that the minimum WCGD corresponds to the canonical (minimum number of components) network realization. Any noncanonical realization cannot have a smaller WCGD and in general will have a larger WCGD. An example will clarify this statement.</u>

Assume a network with only two elements, both resistors, which can deviate from their design value. Then assume $|\varepsilon_1|=|\varepsilon_2| = 0.001$, $S^G_{R_1} = 3$, $S^G_{R_2} = -2$, and thus $S^G_R = 1$ (see Eq. (13)). Thus WCGD = $(.001)(|3| + |-2|) = .005$. However, if the same network could be made with a single resistor, $S^G_{R_1} \equiv S^G_R = 1$, WCGD = $(.001)(1) = .001$. The possibility of a negative sign for one or more of the $S^G_{R_i}$ terms leads to larger WCGD for noncanonical realizations. Without the presence of any negative signs in the $S^G_{R_i}$ functions (or more generally, the $S^G_{e_i}$), the WCGD would be independent of the number of circuit elements, assuming equal ε_i.

III. N3 CHANNEL BAND FILTER DESIGN OBJECTIVES AND THE EQUIRIPPLE APPROXIMATION

For a high-selectivity narrow-band filter, high Q complex poles and imaginary axis zeros are required. The transfer function is of the form

$$G(s) = H \frac{s^{\ell} \prod_{k=1}^{m}\left(s-z_k\right)\prod_{k=1}^{m}\left(s-z_k^*\right)}{\prod_{k=1}^{n}\left(s-p_k\right)\prod_{k=1}^{n}\left(s-p_k^*\right)} \tag{16}$$

where p_k and z_k are complex poles and imaginary zeros, respectively, and p_k^* and z_k^* their complex conjugates. Suppose G(s) is realized by a network and as a result e_i

is a parameter of the network realization. From the definition of $S_{e_i}^{G}$,

$$S_{e_i}^{G} = \sum_{k=1}^{n} \left[\frac{S_{e_i}^{p_k}}{s-p_k} + \frac{S_{e_i}^{p_k^*}}{s-p_k^*} \right] - \sum_{k=1}^{m} \left[\frac{S_{e_i}^{z_k}}{s-z_k} + \frac{S_{e_i}^{z_k^*}}{s-z_k^*} \right] + S_{e_i}^{H} \tag{17}$$

where $S_{e_i}^{H} \triangleq \frac{\partial(\ell n H)}{\partial(\ell n e_i)}$; $S_{e_i}^{p_k} \triangleq \frac{\partial p_k}{\partial(\ell n e_i)}$ is the pole-zero sensitivity p_k with respect to e_i. $S_{e_i}^{z_k}$, $S_{e_i}^{p_k^*}$ and $S_{e_i}^{z_k^*}$ are similarly defined. If we assume that changes in $G(j\omega)$, in the frequency interval where the poles are located, are of primary interest, then all terms in (17) except those involving p_k may be neglected. $S_{e_i}^{H}$ can be disregarded because changes in H are independent of frequency, and hence, can be accommodated by the AGC in the N carrier system. Thus

$$S_{e_i}^{G} \approx \sum_{k=1}^{n} \frac{S_{e_i}^{p_k}}{s-p_k} \tag{18}$$

The sensitivity of p_k is proportional to its Q[11]. Consequently, $S_{e_i}^{G}$ and the attenuation sensitivity, $S_{e_i}^{|G(j\omega)|}$ are proportional to the Q's of the complex poles p_k.

Consider the N3 channel band filter design objectives in terms of the previous discussion.

1. "The attenuation sensitivity is proportional to Q," implies that the design should use the lowest Q poles consistent with specifications.
2. Since the worst-case deviation has been shown, to increase with function complexity, the simplest function G(s) which meets specifications is desired. (Approximation problem.)
3. The mathematical realization should lead to the minimum multiparameter-sensitivity decomposition. (Sensitivity-decomposition problem)
4. The physical network should use the least number of elements, particularly capacitors, and achieve minimum multiparameter sensitivity. (Synthesis problem.)

For this particular study, a channel band specification with a group carrier frequency at 140 KHz was chosen arbitrarily. This is shown in Fig. 4.* The V.S.S.B. filter has a narrow (3.15 KHz) bandwidth and requires 55 dB out-of-band rejection. It also requires an inband flatness of better than ±0.5 dB.

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* The actual N3 channel band frequencies are presently 148-196 KHz. These are somewhat arbitrary, chosen as the best engineering compromise based on crystal and ferrite core design. The present 140 KHz design gives quite sufficient design capability information concerning Tantalum Integrated Circuits.

At the skirt frequencies the allowable tolerance is ±15 Hz. This tolerance is met by the present crystal filter, although it might be able to be relaxed in the actual system to perhaps ±30 or ±40 Hz if sufficient economic inducement accrued.*

For a given response characteristic a number of approximations may meet the nominal specifications. The elliptic filter approximation (equiripple in-band and out-of-band) meets both the first and second design objectives and is most suitable. A typical elliptic filter approximation is given in Fig. 5. The simplest elliptic filter which will satisfy the prescribed channel band filter response and leaves sufficient room for element deviations requires six pairs of complex poles, four pairs of imaginary zeros and two zeros at the origin [12]. The in-band ripple due to the approximation must be much smaller than 0.5 dB to tolerate parameter variations. The theoretical values for the poles and zeros were chosen to correspond to an approximation with ±.01 dB ripple and satisfy the 55 dB rejection characteristic, as shown in Fig. 6. The Q of the highest and lowest pole pairs are approximately 388 and 112, respectively.

IV. MATHEMATICAL DECOMPOSITION OF G(s) AND ITS NETWORK REALIZATION

IV-A Mathematical Decomposition of G(s) Based on Minimum Multiparameter Sensitivity

The voltage transfer function G(s) corresponding the pole-zero configuration of Fig. 6 is

---

* Personal communication, G. Bleisch.

$$G(s) = \frac{s^2\left(s^2+\omega_{o1}^2\right)\cdots\left(s^2+\omega_{o4}^2\right)}{\left[s^2+2\sigma_1 s + \left(\sigma_1^2+\omega_1^2\right)\right]\cdots\left[s^2+2\sigma_6 s + \left(\sigma_6^2+\omega_6^2\right)\right]} . \qquad (19)$$

G(s) has ten finite zeros and twelve poles. We must decompose G(s) into a product of rational realizable biquadratic voltage transfer functions,* each of which contains a pair of complex poles and one or two finite zeros. The decomposition should minimize multiparameter sensitivity. The high Q complex poles should be associated with the zeros so that the distances from the upper half plane complex poles to the zeros are as far pair-distant as possible <u>for each subnetwork function</u>.† The optimum decomposition of G(s) of (19) based on minimum multiparameter sensitivity is

$$\begin{aligned} G(s) = &\left\{\frac{s}{s^2+2\sigma_1 s + \left(\sigma_1^2+\omega_1^2\right)}\right\} \times \left\{\frac{s}{s^2+2\sigma_6 s + \left(\sigma_6^2+\omega_6^2\right)}\right\} \times \left\{\frac{s^2+\omega_{o3}^2}{s^2+2\sigma_2 s + \left(\sigma_2^2+\omega_2^2\right)}\right\} \\ &\times \left\{\frac{s^2+\omega_{o4}^2}{s^2+2\sigma_3 s + \left(\sigma_3^2+\omega_3^2\right)}\right\} \times \left\{\frac{s^2+\omega_{o1}^2}{s^2+2\sigma_4 s + \left(\sigma_4^2+\omega_4^2\right)}\right\} \times \left\{\frac{s^2+\omega_{o2}^2}{s^2+2\sigma_5 s + \left(\sigma_5^2+\omega_5^2\right)}\right\} \end{aligned} \qquad (20)$$

which is shown in Fig. 7.

---

* A rational function of s is a realizable biquadratic voltage transfer function if it has two LHP poles, and the order of the numerator polynomial is not greater than two.

† Reference [11], p. 15.

IV-B Mathematical Decomposition of G(s) Based on Minimum Multiparameter Sensitivity and Functional Adjustment

One of the major differences between making conventional lumped circuits and thin-film circuits is that the former includes the capability of selecting the circuit components within an allowable tolerance, while the Tantalum Integrated Circuit technology allows trimming of element values. Hence, in this memorandum, we suggest that in making thin-film circuits, a novel circuit adjustment technique be employed. Rather than adjusting each circuit component individually to its nominal value, the circuit is functionally adjusted; thus by only adjusting some of the circuit components, preferably resistors, the overall circuit performance is achieved. This is a generalization of a procedure described by Orr [13] for tuning notch network transmission zeros.

Certain passive and active RC networks may be functionally adjusted by resistors alone; i.e., we can accept inexact initial values of the capacitors by trimming the resistors in accordance with a prescribed order of operations, an algorithm. Thus, we eliminate the initial gain tolerances required for capacitors completely; then for a passive RC network, or an active element tolerance is relatively small compared with the tolerances of the passive elements, the gain deviation due to element initial tolerances is equivalent to a frequency scaling on G(s) with a scaling factor equal to $[1 \pm \Delta RC]$. ΔRC denotes the initial network RC-product tolerance which, in fact, is merely the tolerances of the resistors. This general approach to making high performance active RC circuits will be referred to as the <u>functional adjustment</u>. For certain canonical* network realizations of a biquadratic function G(s), an algorithm has been derived to implement functional adjustment. Other canonical networks have not yielded solutions. For instance, the third term in (20),

$$G(s) = \frac{s^2+\omega_{o3}^2}{s^2+2\sigma_2 s + \left(\sigma_2^2+\omega_2^2\right)} \tag{21}$$

can be canonically realized by the active RC network shown in Fig. 8. But, unfortunately, this canonical realization cannot be readily functionally adjusted, because the coefficients of G(s), which involve the elements C_i, R_i and K in both numerator and denominator are complicated functions. Therefore we found need to further decompose it into functions functionally adjustable in practice. If we decompose (21) into two functions

$$G(s) = \frac{s^2+\omega_{o3}^2}{s^2+2\sigma_2 s + \left(\sigma_2^2+\omega_2^2\right)}$$

---

* "canonical" means "with a minimum number of elements" when applied to network theory.

$$= \frac{s^2+\omega_{o3}^2}{s^2+4\omega_{o3}s+\omega_{o3}{}^2} \cdot \frac{s^2+4\omega_{o3}s+\omega_{o3}^2}{s^2+2\sigma_2 s + \left(\sigma_2^2+\omega_2^2\right)} \tag{22}$$

this expression can be realized by a functional adjustable twin-tee network in cascade with functional adjustable active RC network. Applying the same decomposition for the fourth, fifth and sixth terms of (20), G(s) is redecomposed as

$$G(s) = \left\{\frac{s^2+\omega_{o3}^2}{s^2+2\sigma_2 s + \left(\sigma_2^2+\omega_2^2\right)}\right\} \times \left\{\frac{s^2+\omega_{o4}^2}{s^2+2\sigma_3 s + \left(\sigma_3^2+\omega_3^2\right)}\right\} \times \left\{\frac{s^2+\omega_{o1}^2}{s^2+2\sigma_4 + \left(\sigma_4^2+\omega_4^2\right)}\right\}$$

$$\left\{\frac{s^2+\omega_{o2}^2}{s^2+2\sigma_5 s + \left(\sigma_5^2+\omega_5^2\right)}\right\} \times \left\{\frac{s}{s^2+2\sigma_1 s + \left(\sigma_1^2+\omega_1^2\right)}\right\} \times \left\{\frac{s}{s^2+2\sigma_6 s + \left(\sigma_6^2+\omega_6^2\right)}\right\}$$

$$= \left\{\frac{s^2+\omega_{o1}^2}{\underbrace{s^2+4\omega_{o3}s+\omega_{o3}^2}_{\leftarrow \text{TWIN-TEE} \rightarrow}} \; \frac{s^2+4\omega_{o1}s+\omega_{o1}^2}{\underbrace{s^2+2\sigma_2 s + \left(\sigma_2^2+\omega_2^2\right)}_{\leftarrow \text{ACTIVE RC} \rightarrow}}\right\} \times \left\{\frac{s^2+\omega_{o4}^2}{s^2+4\omega_{o4}s+\omega_{o4}^2} \; \frac{s^2+4\omega_{o4}s+\omega_{o4}^2}{s^2+2\sigma_3 s + \left(\sigma_3^2+\omega_3^2\right)}\right\}$$

$$\left\{\frac{s^2+\omega_{o1}^2}{s^2+4\omega_{o1}s+\omega_{o1}^2} \; \frac{s^2+4\omega_{o1}s+\omega_{o1}^2}{s^2+2\sigma_4 s + \left(\sigma_4^2+\omega_4^2\right)}\right\} \times \left\{\frac{s^2+\omega_{o2}^2}{s^2+4\omega_{o1}s+\omega_{o1}^2} \; \frac{s^2+4\omega_{o2}s+\omega_{o2}^2}{s^2+2\sigma_5 s + \left(\sigma_5^2+\omega_5^2\right)}\right\}$$

$$\left\{\frac{s}{s^2+2\sigma_1 s + \left(\sigma_1^2+\omega_1^2\right)}\right\} \times \left\{\frac{s}{s^2+2\sigma_6 s + \left(\sigma_6^2+\omega_6^2\right)}\right\} \qquad (23)$$

$$= G_1(s)G_2(s)\ldots G_{10}(s)$$

Refer to Fig. 6 for values of σ_i, ωo_i and ω_i.

G(s) which can be synthesized by a cascade of ten networks, each of which realizes a $G_i(s)$ as shown in Fig. 9. This is an optimum decomposition of G(s) based on minimum multiparameter sensitivity and functional adjustment which fulfills the third design objective.

IV-C Network Realization of G(s)

Having decomposed G(s) into ten biquadratic functions, it is appropriate to synthesize each of them by a passive or active RC network. Many active RC synthesis procedures may be used, involving operational amplifier, negative impedance converter or gyrator methods. The gyrator appears not to be the best active element above 100 KHz [17]. Three advantages exist for using finite-gain operational amplifier methods, instead of negative impedance converter methods, in the realization of G(s):

1. Operational amplifiers can be made somewhat more stable (stable to 1 part in 10^5) than negative impedance converters (NIC's).
2. The NIC realizations are not as readily amenable to functional adjustment as certain operational amplifier realizations. Furthermore, NIC realizations appear, in general, to have higher

multiparameter sensitivities than the corresponding operational amplifier realizations.

3. NIC realizations usually contain an excess number of elements. Operational amplifier realizations have fewer; quite often, they are canonical.

The network realizations of the ten stages are shown in Fig. 10-13. For the four twin-tee networks, $RC = 1/2\pi\omega_{oi}$, $C = 1000$ pF is appropriate for thin film realizations; the corresponding resistor values are shown in Fig. 10. The four active bridge-tee networks [14] and the two high Q active RC networks [15] have been drawn from the literature to have minimum WCGD in light of tantalum integrated circuit technology. Since the realization of each stage is canonical, the over-all realization of G(s) of (23) contains minimum numbers of circuit elements; this fulfills the fourth design objective. The total number of elements used in the design is summarized in Table 1.

Table 1 - NUMBER OF CIRCUIT ELEMENTS

Elements	Number
Resistors	26
Capacitors	26
Operational amplifiers	6
Summing circuits	2
Buffer amplifiers	9

V. TOLERANCE STUDY

In the previous section, the channel band nominal characteristic was achieved by an active RC network, based on the minimum multiparameter sensitivity and functional adjustment considerations. The prescribed element values are achieved only approximately in practice; element tolerances due to trimming, temperature, humidity and aging are inevitable. Consequently, any practical design must tolerate small changes in element values. In Tantalum Integrated Circuits, there are three major tolerances: initial tolerance (including corrective trimming), temperature tolerance, and aging and humidity tolerance.

The allowable overall gain tolerance bounds of the N3 channel band filter were shown in Fig. 4. The inband tolerance is ±0.5dB, and the allowable skirt tolerance is nominally ±15 Hz. In what follows, we shall calculate the gain deviation with state of the art values for each of the above element tolerances to determine how closely the gain deviation can be held to the specified bounds. The assumed element tolerances are tabulated in Table 2. Their character is given in Table 3.

Table 2 - ASSUMED ELEMENT TOLERANCES

Variation Parameter	Initial Tolerance	Temperature Tolerance	Aging Tolerance
Resistor R	±200 PPM (.02%)	RC-Product	+200 PPM (.02%)
Capacitor C	±10,000 PPM* (1%)	≈ ±20 PPM/°C (.002%)	±1000 PPM (.1%)
Gain of Op. Amp. K	±10 PPM (.001%)	Negligible	

Table 3 - CHARACTER OF ELEMENT TOLERANCES

Initial Tolerance	R	Random
	C	
	K	
Temperature Tolerance	RC-Product	Random
Aging Tolerance	R	Tracking
	C	Random
	K	Random

V-A Initial Tolerance

In Section IV, ten networks were used in cascade to realize the channel band characteristic. This includes four twin-tee networks, two active feedback networks and four active bridged-T networks (Fig. 14) which are all functionally adjustable. Now we shall study the initial tolerance of each network after functional adjustment.

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* Except for several critical values, with initial tolerances ±0.1%, as discussed later in this section.

1. Initial Tolerance Functional Adjustment of the Passive Networks

The tuning of a tantalum thin-film twin-tee notch network by precise and controlled trimming of resistor values was studied by Orr [13]. He found that a twin-tee notch network could be adjusted for the notch frequency within ±0.02% of the desired value. Applying his tuning procedure to the four twin-tee networks of Fig. 10 the maximum deviations of the notch frequencies of the four networks should be within ±0.02% of their desired values as shown in Fig. 15. From these eight deviation curves we may find the worst-case upper and lower bounds of the over-all gain deviations (in dB) due to the initial tolerances of the four twin-tee networks. The in-band worst-case gain deviation after absolute gain adjustment is found to be ±0.14 dB.* The procedure for finding the worst-case upper and lower bounds of the overall gain deviations (as well as the computation by the digital computer) is given in Appendix A. To adjust the notch frequency within ±0.02%, we must adjust the initial precision of the network RC product within ±0.02%, equivalent to a frequency scaling of the network function. The over-all gain deviation (due to the

---

* Note that the .14 dB gain deviation figure may be decreased by mechanization of finer resistor trimming (realizing each resistor by two series elements, one of which is much smaller than the other). Bridge accuracy is the only eventual limit.

initial tolerances of the four twin-tee networks) at the skirt frequencies will be discussed at the end of the following section.

2. Initial Tolerance Functional Adjustment of Active RC Networks

Now we extend the idea of tuning the twin-tee notch frequency by adjusting resistor values to the broader concept of functional adjustment* of other passive RC networks as well as *active* RC networks. To illustrate the method of functional adjustment of an active RC network, we use the network of Fig. 13, which is redrawn in Fig. 17(a). The operational feedback amplifier has overall gain K:

$$K = \frac{}{1 + \frac{R_i}{R_f} A} \approx \frac{R_f}{R_i}; \quad A \gg 100 \tag{24}$$

This active network realizes a pair of complex poles and a zero at the origin (Fig. 17(b)), resulting in a frequency response (Fig. 17(c)) which peaks at ω_c. The voltage transfer function is

$$G_0(s) = \frac{R_2C_2s}{R_1R_2C_1C_2s^2 + [R_1(C_1+C_2) + (1-K)R_2C_2]s+1}$$

$$= \frac{1}{R_1C_1} \frac{s}{s^2 + \frac{R_1(C_1+C_2) + (1-K)R_2C_2}{R_1R_2C_1C_2} s + \frac{1}{R_1R_2C_1C_2}}$$

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* Here the word adjustment is used instead of tuning because tuning in general means tuning a resonant or anti-resonant frequency of a selective network, but adjustment may mean to adjust any particular desired characteristic of a network.

$$= H \frac{s}{s^2+2\sigma_c s+\omega_n^2} \quad (25)$$

where $\omega_n^2 = \sigma_c^2 + \omega_c^2$. This network can be functionally adjusted by three resistors: R_1, R_2 and R_f.

In the deviation studies which follow we will define both the nominal (design) element values and the initial and trimmed element values. The actual circuit values that result from trim adjustment will be superscripted "a"; i.e. R_1^a will be the trimmed value of R. Untrimmed (initial) values will be designated with a superscript "i"; i.e., C_1^i, or R_1^i. Since capacitors will not be adjusted, the initial capacitor values will also serve as the actual circuit values.

Let $\tau = R_1 (C_1+C_2)$. The functional adjustment follows from (25).

1. Measure C_1^i and C_2^i. Anodize R_1^i to R_1^a such that $R_1^a \left(C_1^i + C_2^i\right) = \tau^a = \tau \pm 0.02\%$ (since R_1^a may be trimmed with 0.02% or R_1).
2. From an oscillographic presentation of $|G(j\omega)|$, anodize R_2^i such that $\left(\omega_c^a\right)^2 = \omega_c^2 \pm 0.02\%$ or $\omega_c^a = \omega_c \pm .01\%$. Thus $\omega_n^2 = \omega_c^2 \left(1 + \frac{1}{4Q2}\right) \cong \omega_c^2$ and ω_n is achieved within .01%.
3. The third anodization is used to critically adjust the value of σ_c and thus Q. This requires a careful exposition of the procedure, with numerical illustrations

since 0.02% errors in K reflect far larger errors in σ_c and may possibly result in oscillation. It is necessary to examine the expression for σ_c, together with nominal numerical values for each term.

$$\sigma_c = \frac{1}{2}\left[\frac{R_1(C_1+C_2) + (1-k)\ R_2C_2}{R_1R_2C_1C_2}\right]$$

$$= \frac{\tau}{2\omega_n^2}\left[1 - \frac{R_2C_2}{\tau}\ (K-1)\right]$$

For the circuit of Eq. (25),

$$\frac{R_2C_2}{\tau} = \frac{0.88183588\times 10^{-3}}{0.88198231\times 10^{-5}}$$

$$= 0.99983398\times 10^{2}.$$

Also K = 1.0099983 and thus (K-1) = .0099983. Thus a ±.02% error in K (i.e., ±.0020199) corresponds to a ±2% error in (K-1) which is critical in constructing this algorithm. This arises, fundamentally, because active RC synthesis procedures use the active transmittance coefficient to reduce σ_c by constructing it from the differences of two <u>almost equal</u> numbers.

Furthermore, examining the term

$$\left[1 - \frac{R_2C_2}{\tau}(K-1)\right] = A$$

we note that a 2% error in (K-1) corresponds to a far larger error in A. Specifically if $\frac{R_2C_2}{\tau}(K-1) = 0.9996639 \pm 2\%$ $= .9996639 \pm .019993$, then $A = .0003361 \pm .0199$. This implies σ_c has the same unreasonable tolerance.

Thus it is necessary to anodize the resistors (R_f and R_i) to a far better tolerance than .02%. Such a procedure will be demonstrated shortly. Assume that K may be trimmed to within 1 part in 10^8. Then (1-K) is accurate within 1 part in 10^6, and

$$\left[\frac{R_2C_2}{\tau}(K-1)\right] = .0006639 \pm .0001\%$$
$$= .9996639 \pm .000000999$$

Hence $A = .0003361 \pm .000000999$

$= .0003361 \pm .29\%$

Thus σ_c and Q are adjustable within $0.29\% \pm .04\% = .33\%$, since $\sigma_c = \frac{A\tau}{2\omega_{n_o}^{\;2}}$. Trimming K may be accomplished by successively trimming the input resistor R_i and the feedback resistor R_f whose ratio R_f/R_i determines K. R_f and R_i are each realized by three resistors in series; typical values will be 10^6 ohms, 10^3 ohms and 10 ohms. Initial trimming is accomplished on the largest resistors, which are monitored. Subsequent trimming on the smaller resistors is carried out while monitoring K or (K-1) with a digital voltmeter.

After functional adjustment of the two networks of Fig. 13 using the above procedure, we may calculate the worst-case in-band gain deviation due to the initial tolerance of the two networks as follows: let G′(s) be the voltage transfer function of the network after function adjustment:

$$G'(s) = H' \quad s^2 + 2\sigma_c' s + \left(\omega_n'\right)$$

where σ_c' and ω_c' are the real and imaginary parts of the complex pole after functional adjustment, and $\omega_n' = \sqrt{(\sigma_c')^2 + (\omega_c')^2}$. Then the worst-case in-band gain deviation (WCD) may be calculated by

$$WCD = \frac{G'(j\omega_n') - G_o(j\omega_n)}{G_o(j\omega_n)} = \frac{H' \dfrac{1}{\sigma_c'} - H \dfrac{1}{\sigma_c}}{H \cdot \dfrac{1}{\sigma_c}} \tag{26}$$

Since $H' \approx H$ and $\sigma_c' = (1 \pm 0.33\%)\ \sigma_c$, (26) is found to be

$$WCD \cong \left| \frac{1}{1 \pm 0.0033} - 1 \right| = \pm 0.0033 = \pm .06\text{dB} \tag{27}$$

The remaining four active bridged-tee networks can be similarly functionally adjusted. The worst-case in-band gain deviation due to the initial tolerance of each of these four networks is calculated to be ±.08dB, as shown in Appendix B. Appendix B also includes functional adjustment algorithms for these networks. The assumed initial tolerance of an operational amplifier is ±10 ppm which is much smaller than the resistor and capacitor tolerances (see Table 2). In this case the change in RC product is still nearly equivalent to a frequency scaling of the network function. Observe that since we accept all the capacitor values and use resistor anodization to adjust the network functionally, the initial precision of RC products is the same as the assumed initial tolerance of resistors, i.e. ±0.02%.* Thus the overall gain deviation due to the ten networks at the skirt frequencies is 0.02%* f_s, where f_s is the skirt frequency, or about ±28 Hz. This may be brought within the prescribed constraint, ±15 Hz by an initial resistor tolerance of .01%. The gain deviation due to the initial tolerances of the ten networks after functional adjustment is summarized and tabulated in Table 4, and is shown in Fig. 18.

The deviations associated with the six active networks exist primarily within the region of each of their

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* This can probably be improved to .01% by careful fabrication.

dominant poles, and do not cause significant deviations at the frequencies associated with the poles of the remaining active networks. Hence there is little interaction of deviations between active networks; i.e., the maximum gain deviation over the in-band region due to the pole deviations is approximately that due to a single pole, and at any other in-band frequency is well less than that due to any pair of poles. Hence the numerical values of Table 4.

Table 4 - THE GAIN DEVIATION DUE TO THE INITIAL TOLERANCES OF THE TEN NETWORKS

Gain Deviation Networks	Max. Gain Deviation at in-band frequencies	Gain Deviation at skirt frequencies
Four twin-tee networks (total)	±0.14 dB	
Two Active feedback networks, each	±0.06 dB ±0.28 dB	≃ ±28 Hz (or 14 Hz see text)
Four Active bridged-tee networks, each	±0.08 dB	

These figures can be improved to nearly meet N3 requirements by serially adjusting resistor pairs as discussed on p. 25 instead of single resistors, until limited by bridge accuracy.

V-B <u>Temperature Tolerance Study</u>

The temperature tolerance of the RC product of a tantalum thin-film RC network may be made as little as ±20 ppm/°C, by making the temperature coefficient of the resistors approximately equal but opposite in sign to that of the capacitors. With the resistor feedback operational amplifier shown in Fig. 17, whose closed-loop gain K was expressed in (24), the gain is made essentially independent of temperature variation because the temperature coefficients of the two resistors R_i and R_f track. The dominant temperature tolerance of such an active RC network is the tolerance of the RC product, which is again equivalent to a frequency scaling of the network function. Thus the gain deviation at the skirts is approximately equal to ±20 ppm/°C × 140 KHz = ±2.8 Hz/°C. A skirt tolerance of ±15 Hz (30 Hz) corresponds to an allowable temperature variation of ±6°C (12°C) as shown in Fig. 19. There is no significant in-band gain deviation due to temperature tolerance; the ±0.01 dB deviation indicated (Fig. 19) is the ripple due to the equiripple function use in the original synthesis procedures.

V-C <u>Aging Tolerance Study</u>

The best aging tolerances of tantalum resistors and capacitors are assumed to be +200 ppm and ±1000 ppm, respectively. The aging tolerance of an operational amplifier is negligible. Based on these assumptions, the frequency deviation at the skirts is about ±1200 ppm × 140 KHz = 168 Hz and the gain deviation at the in-band frequencies is about ±0.25 dB shown in Fig. 20.

To summarize the above tolerance studies, it is found that by using present state-of the art Tantalum Integrated Circuit element tolerances, we can readily meet the in-band gain tolerance requirement, and approximate the skirt gain tolerance requirement except for the capacitor aging factor in four critical circuits. An order of magnitude improvement in tantalum capacitor aging properties appears necessary to be able to use these elements in meeting high selectivity specifications. Capacitors with aging tolerances of 100 ppm, required for the four critical circuits which realize the two imaginary zeros ω_{o2} and ω_{o3} and the two highest Q pole pairs p_1 and p_6 associated with the skirt characteristics (Table 5) may be met by polystyrene or ceramic capacitors for successful circuit implementation. See the footnote associated with Table 5.

Table 5 - ELEMENT VALUES OF THE FOUR CRITICAL CIRCUITS

CIRCUIT	ELEMENT	NUMBER
Twin-tee No. 1	R = 1142.69 pF	3
	* C = 1000 pF	3
Twin-tee No. 2	R = 1102.96 Ω	3
	* C = 1000 pF	3
Active Feedback No. 1	K = 1.00999	1
	R = 100 Ω	1
	R = 10K Ω	1
	* C_1 = 14.6 pF	1
	C_2 = 0.08 µF	1
Active Feedback No. 2	K = 1.00999	1
	R = 100 Ω	1
	R = 10K Ω	1
	* C_1 = 14.6 pF	1
	C_2 = 0.08803 µF	1

* 100 ppm capacitor aging tolerance is required. Appliqued polystyrene or ceramic capacitors are suggested, as possible candidates for this application. Masland (MM 66-2623-24) has tested monolethic ceramic capacitors which have demonstrated an aging stability of better than .02%.

VI. CONCLUSION

An optimum active RC design of an N3 channel band filter, has been presented. This includes an optimum decomposition of the network function obtained from an equiripple in-band and out-band approximation of the filter characteristic, based on the minimum multiparameter sensitivity. Functional adjustment - a method of obtaining active RC tantalum thin-film integrated circuits with minimum multiparameter sensitivity - has been introduced. The design contains a minimum number of circuit elements with respect to the network function and meets design objectives. The gain deviation due to the three major tolerances - initial, temperature, and aging tolerances - has been studied in great detail. It is concluded that:

1. Tantalum active RC networks meet N3 filter requirements at lower carrier frequencies without field adjustment to correct for capacitor aging. This might be accomplished by pregrouping; i.e., modulating of the voice channel to 24-40 KHz, removal of one side-band with an active RC filter, then modulation to the N3 line frequencies (36-132 KHz and 172-268 KHz).
2. At 140 KHz, an initial RC product tolerance of $\pm 0.01\%$ and a temperature variation $< 10°C$ are

required and can be achieved. Tantalum capacitor aging, the dominant cause of the skirt and gain deviation, requires an order of magnitude improvement for an <u>all tantalum</u> passive element technology to be feasible. Appliqued polystyrene or ceramic capacitors should afford appropriate substitutions in critical circuits to permit the realization of excellent highly selective channel band filters as active RC networks.

SCLee/RW

S. C. LEE

R. W. WYNDRUM, JR.

MH-2623-$\frac{\text{SCL}}{\text{RWW}}$-MG

Att.
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Appendix A & B
Figures 1 through 20

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APPENDIX A

THE WORST-CASE IN-BAND GAIN DEVIATION DUE TO THE FOUR TWIN-TEE NETWORKS AFTER FUNCTIONAL ADJUSTMENT

The frequency response of a twin-tee network with a notch frequency at ω_o can be expressed as [16]

$$G(j\omega) = \frac{\left(\frac{\omega_o}{\omega} - \frac{\omega}{\omega_o}\right)}{\left(\frac{\omega_o}{\omega} - \frac{\omega}{\omega_o}\right) + j4} = \frac{A_o(\omega)}{A_o(\omega) + j4} \tag{A-1}$$

Then

$$|G(j\omega)| = \frac{A_o(\omega)}{\sqrt{A_o(\omega) + 16}} \tag{A-2}$$

Define $|G(j\omega)|$ to be the gain deviation of the twin-tee network with the notch frequency at ω_{o_i}, and $|G_i'(j\omega)|$ and $|G_i''(j\omega)|$ to be the gain deviation of the twin-tee network whose notch frequency is at 0.9998 ω_{o_i} and 1.0002 ω_{o_i} and 1.0002 ω_{o_i}, respectively, i = 1, 2, 3 and 4. Furthermore, let

$$D_i(\omega) = \frac{|G_i'(j\omega)| - |G_i(j\omega)|}{|G_i(j\omega)|} \tag{A-3}$$

$$E_i(\omega) = \frac{G_i''(j\omega) - G_i(j\omega)}{|G_i(j\omega)|} \tag{A-4}$$

APPENDIX B

FUNCTIONAL ADJUSTMENT OF THE ACTIVE BRIDGED-TEE NETWORKS

(I) The voltage transfer function of the active bridged-tee network of Fig. 11 is

$$G_I(s) = \frac{K[R_1R_2C_1C_2s^2+C_2(R_1+R_2+R_3)s+1]}{[R_1R_2C_1C_2+(1-K)C_1C_2]s^2 + [(R_1+R_2+R_3)C_2+(1-K)R_1C_1]s+1}$$

$$= \frac{K[a_1s^2+b_1s+1]}{(a_1+a_2)s^2+(b_1+b_2)s+1} = \frac{K[a_1s^2+b_1s+1]}{As^2+Bs+1} \qquad \text{(B-1)}$$

where $a_1 = R_1R_2C_1C_2$, $a_2 = (1-K)\ C_1C_2$, $b_1 = (R_1+R_2+R_3)C_2$ and $b_2 = (1-K)R_1C_1$. The four terms a_1, a_2, b_1 and b_2 can be adjusted by the four resistors R_1, R_2, R_3 and R_f of the operational amplifier.

The procedure of functional adjustment of this network is as follows:

(1) Assume that the initial tolerances of C_1 and C_2 are within ±1%. Adjust R_f of the operational amplifier so that $(1-K)C_1C_2 = a_2 \pm (1-K \pm 0.02\%)/(1-K) = a_2 \pm 0.0225\%$. Notice that for a ±2% tolerance in C_1C_2, $a_2 \pm 2\%$ adjustment of (1-K) from its nominal is needed to compensate this tolerance. Since K is 8.942 and 8.9556 for the two networks (Fig. 11) which is nearly nine times larger than unity, any tolerance in $C_1 \cdot C_2$ less than or equal to ±2% can always be adjusted by K.

i = 1, 2, 3 and 4. A close observation of the twelve curves in Fig. 15 tells us that the worst-case upper and lower bounds of the in-band gain deviation due to the four functionally adjusted networks can be calculated from

$$\text{The worst-case upper bound} = D_1(\omega) + D_2(\omega) + E_3(\omega) + E_4(\omega) \tag{A-5}$$

$$\text{The worst-case lower bound} = E_1(\omega) + E_2(\omega) + D_3(\omega) + D_4(\omega) \tag{A-6}$$

A computer program has been written and used to calculate the worst-case upper and lower bounds which are plated in Fig. 16. Using absolute gain adjustment, we find that the worst-case gain deviation is ±0.46 dB - (±0.32 dB) = ±0.14 dB.

(2) Adjust R_1 so that $(1-K)R_1C_1 = b_2 \pm 0.02\%$

(3) Adjust R_2 so that $R_1R_2C_1C_2 = a_1 \pm 0.02\%$

(4) Adjust R_3 so that $(R_1+R_2+R_3)C_2 = b_1 \pm [R_1+R_2 + (R_3 \pm 0.02\%)]/[R_1+R_2+R_3] = b_1 \pm$ Here we assumed that the deviation of C_2, R_1 and R_2 is within the adjustability of R_3

(5) If the amount of adjustment required by R_3 is beyond the adjustability of R_3, we may adjust B by R_f if B is larger than its nominal, or by R_2 if B is less than its nominal, with an aid of an oscilloscope, where R_f and R_2 can always be adjusted by an amount that is needed.

It is important to notice that after functional adjustment, the tolerance of B should be within ±0.02%. The worst case tolerance of A can be found as follows: since $R_1R_2 >> (1-K)$, $C_1C_2R_1R_2 >> (1-K)C_1C_2$. Therefore the tolerance of a_1 is nearly to be the tolerance of A. In step 5, for the case that B is larger than its nominal varying K a small amount to adjust B almost does not affect the value of A. However, for the case that B is less than its nominal and the amount of adjustment needed by $(R_1+R_2+R_3)$ beyond the value of R_3, and assuming that C_2 has a tolerance of ±1%, the worst-case tolerances of R_1 and R_2 are ±1%. The tolerance found in Step (3) after functional adjustment was ±0.02%. Therefore the total worst-case tolerance of $R_1R_2C_1C_2$ becomes ±1.02% which is the worst-case tolerance of A.

Let $G_I'(s)$ be the voltage transfer function of the network after functional adjustment

$$G_I^1(s) = \frac{K'[a_1's^2+b_1's+1]}{A's^2+B's+1} = \frac{H's}{s^2 + \frac{B'}{A'}s + \frac{1}{A'}} \qquad \text{(B-2)}$$

where the "prime" indicates the quantity after functional adjustment. Let $p_c' = -\sigma_c^1 + j\omega_c'$ and $\omega_n' = \sqrt{(\sigma_c')^2 + (\omega_c')^2}$. Then

$$G_I'(j\omega_n') = \frac{H'(j\omega_n')}{j\,\frac{B'}{A'}\,\omega_n'} = \frac{H'(j\omega_n')}{j\,\frac{B \pm 0.02\%}{A \pm 1.02\%}\,\omega_n'} \qquad \text{(B-3)}$$

Accordingly, we find that

$$G_I(j\omega_n) = \frac{H(j\omega_n)}{j\,\frac{B}{A}\,\omega_n} \qquad \text{(B-4)}$$

Since $H(j\omega_n) \approx H\ (j\omega_n')$, and $\omega_n \approx \omega_n'$ the worst-case in-band gain deviation is

$$(WCD)_I = \frac{G_I'(j\omega_n') - G_I(j\omega_n)}{G_I(j\omega_n)} = \frac{\frac{B\pm0.02\%}{A\pm1.02\%} - \frac{B}{A}}{\frac{B}{A}} \qquad \text{(B-5)}$$

$$= \begin{matrix}+0.0105\\-0.0103\end{matrix}\ (\pm\ 0.08\ \text{dB})$$

(II) The voltage transfer function of the active bridged-tee network of Fig. 12 is

$$G_{II}(s) = \frac{K\{C_1C_2C_3R_1R_2s^2 + [(C_1+C_2)C_3 + C_1C_2]R_1s + C_3\}}{C_1C_2C_3R_1R_2s^2 + \{[(C_1+C_2)C_3+C_1C_2]R_1+(1-K)R_2C_2C_3\}s + [C_3+(1-K)C_2]}$$

$$= \frac{K\{a_os^2+d_os+e_o\}}{a_os^2+b_os+c_o} \qquad \text{(B-6)}$$

The procedure of functional adjustment of this network is as follows:

(1) Assume that the initial tolerances of C_1, C_2 and C_3 are within ±1%, and $C_1 \doteq C_2$.

(2) Adjust R_1 so that $R_1[(C_1+C_2)C_3+C_1C_2] = d_o \pm 0.02\%$.

(3) Adjust R_2 so that $C_1C_2C_3R_1R_2 = a_o \pm 0.02\%$.

(4) Adjust R_f of the operational amplifier so that $[(C_1+C_2)C_3+C_1C_2]R_1+(1-K)R_2C_2C_3 = b_o \pm 0.02\%$.

(5) $C_3 \gg (1-K)C_2$, so $C_3 + (1-K)C_2 = c_o \pm 1\%$.

By the same reasoning as in (I), the worst-case in-band deviation is calculated to be

$$(WCD)_{II} = \frac{G'_{II}(j\omega_n') - G_{II}(j\omega_n)}{G_{II}(j\omega_n)} = \frac{\dfrac{b_o \pm 0.02\%}{a_o \pm 0.02\%} - \dfrac{b_o}{a_o}}{\dfrac{b_o}{a_o}}$$

$$= \pm 0.0004 \ (0.008 \text{ dB})$$

Therefore $(WCD)_{I+II} = (WCD)_I \vee (WCD)_{II} = \pm 0.08$ dB.

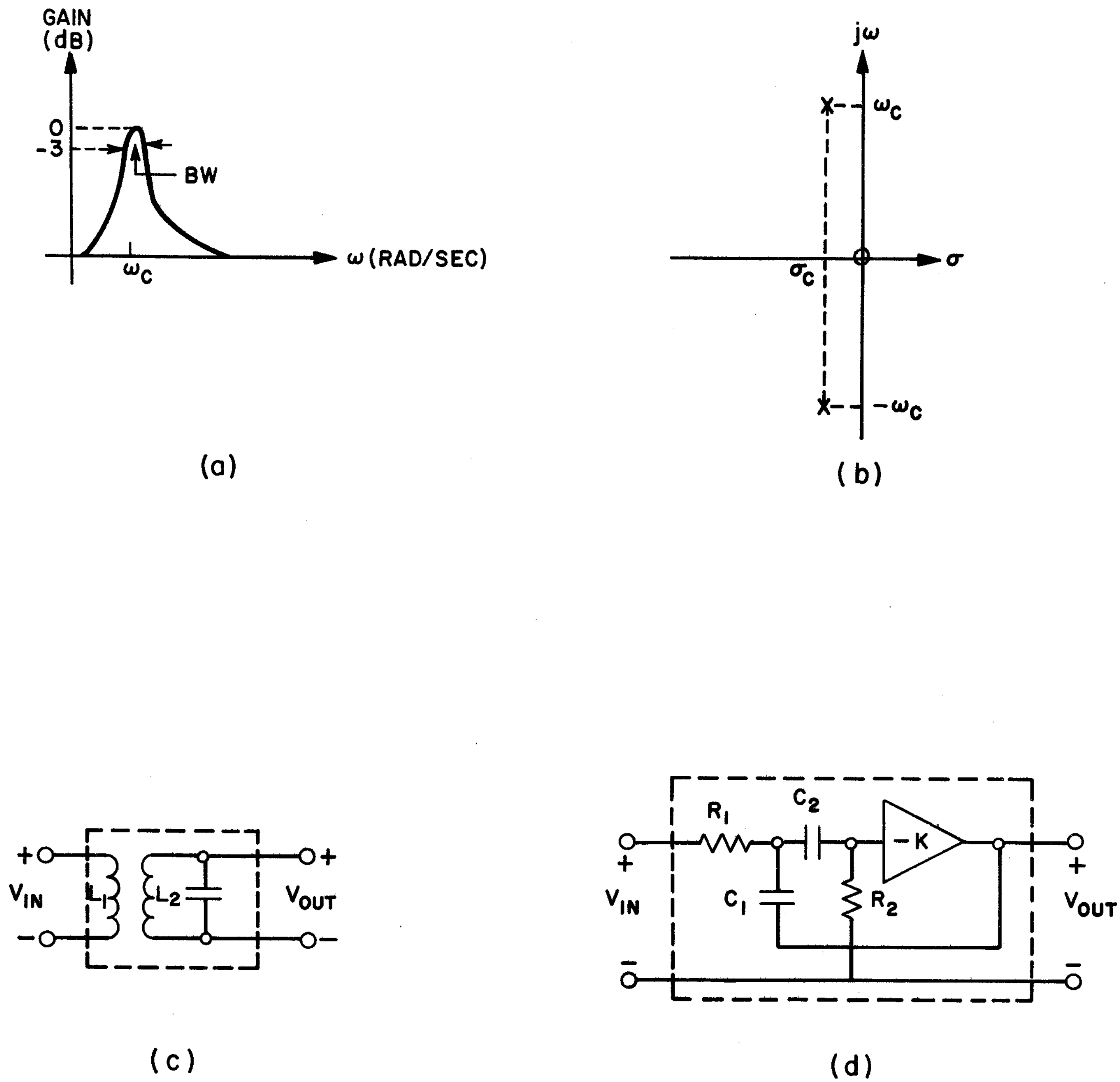


FIG. I (a) A BAND-PASS FREQUENCY RESPONSE CHARACTERISTIC

(b) THE POLE-ZERO REPRESENTATION OF (a)

(c) A CONVENTIONAL LC TUNING CIRCUIT REALIZATION

(d) AN INDUCTORLESS ACTIVE RC REALIZATION USING AN OPERATIONAL AMPLIFIER

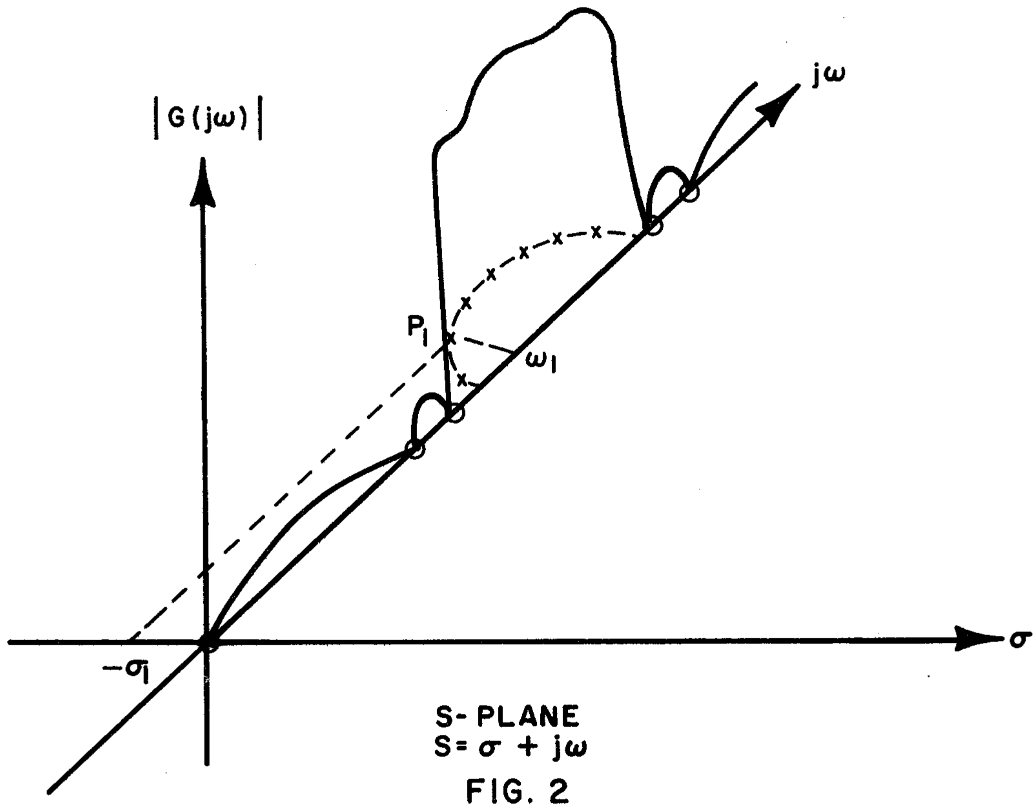


FIG. 2

A POLE - ZERO CONFIGURATION TO APPROXIMATE
A BAND-PASS FREQUENCY RESPONSE

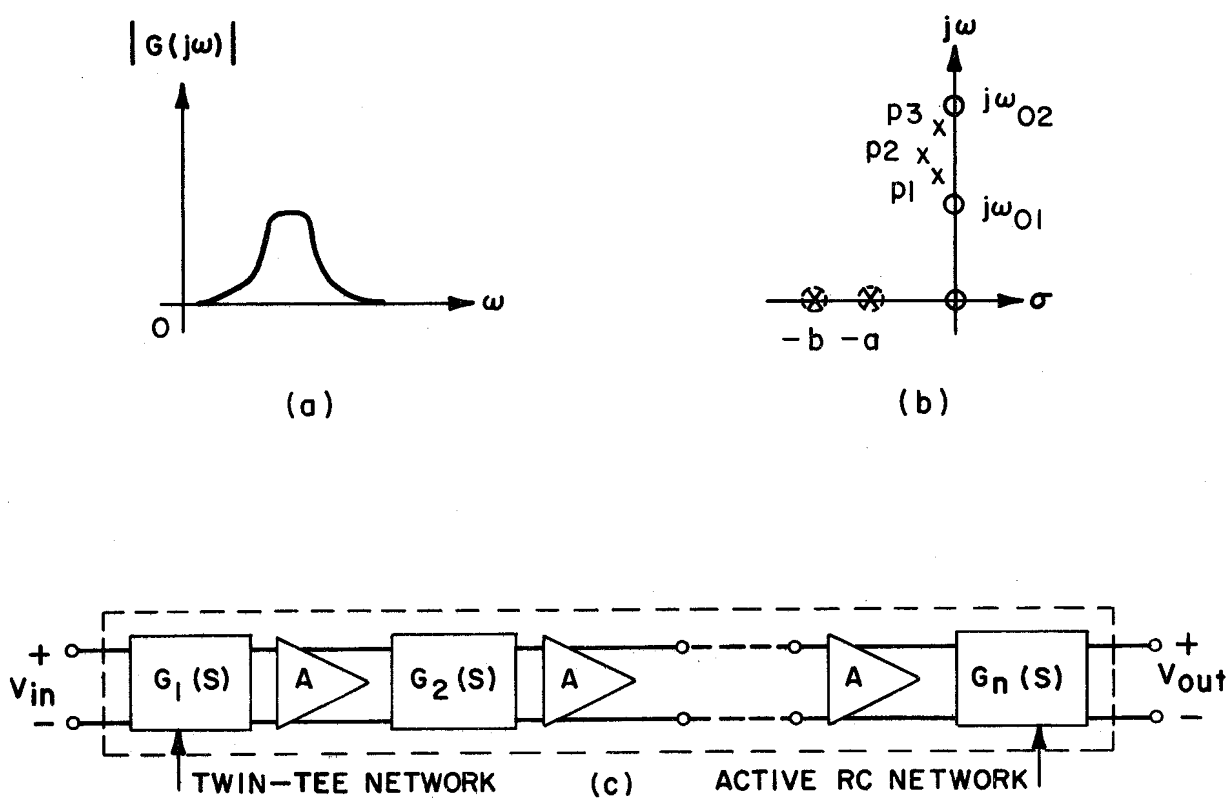


FIG. 3

(a) A BAND-PASS FREQUENCY RESPONSE CHARACTERISTIC
(b) THE POLE-ZERO CONFIGURATION OF THE FREQUENCY RESPONSE
(c) THE CASCADE SYNTHESIS

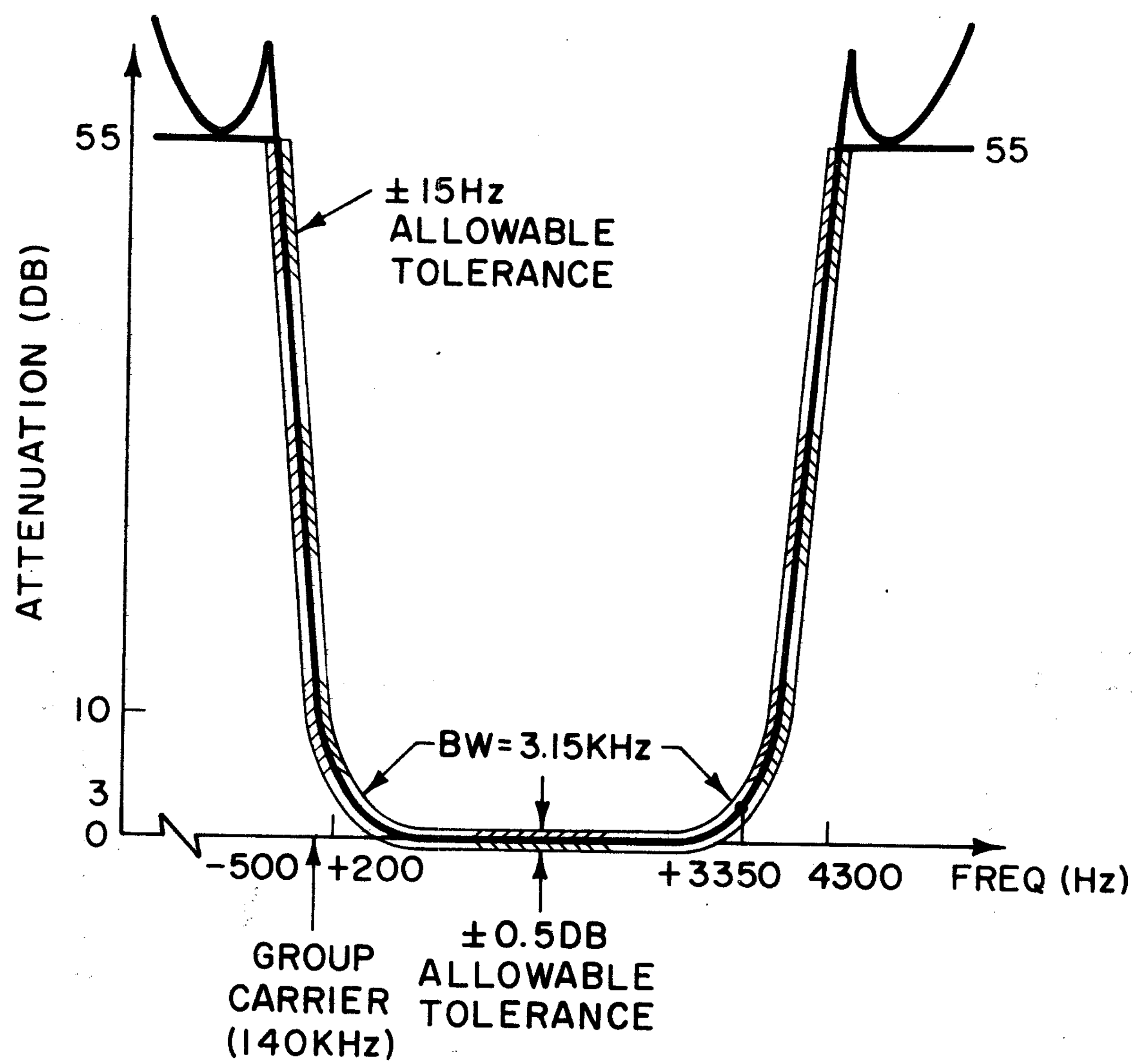


FIG. 4 N3 CHANNEL BAND FILTER SPECIFICATION

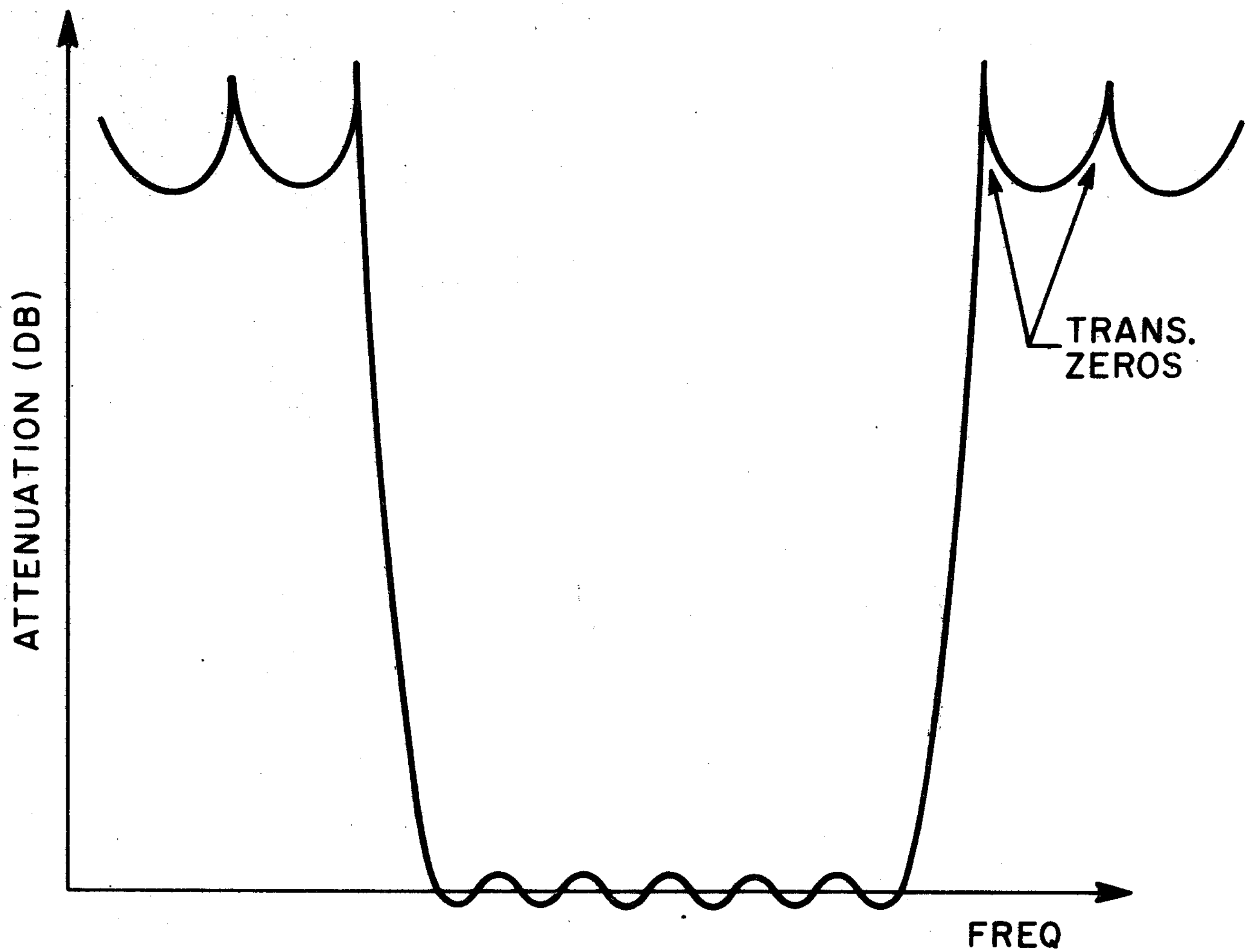
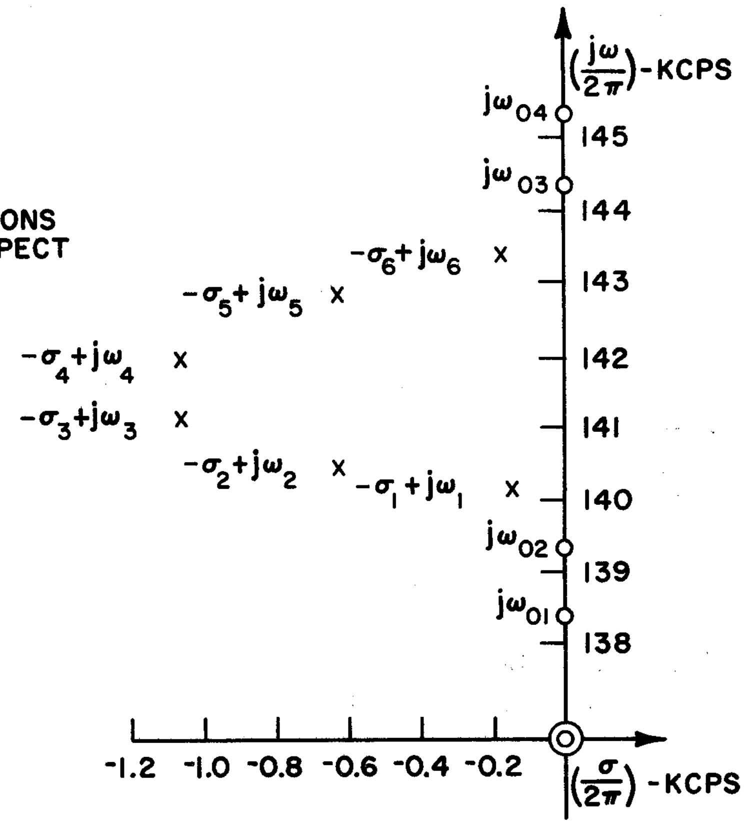


FIG. 5 A TYPICAL EQUIRIPPLE IN-BAND AND OUT-BAND APPROXIMATION

NOTE:
1. NOT DRAWN TO SCALE
2. POLE AND ZERO POSITIONS NORMALIZED WITH RESPECT TO 2π



$\omega_{01} = 2\pi \times 138{,}446.43$

$\omega_{02} = 2\pi \times 139{,}280.13$

$\omega_{03} = 2\pi \times 144{,}296.73$

$\omega_{04} = 2\pi \times 145{,}165.72$

$\sigma_1 = 2\pi \times 180.48138$	$\omega_1 = 2\pi \times 140{,}058.19$	$Q_1 = 388.01285$
$\sigma_2 = 2\pi \times 625.04691$	$\omega_2 = 2\pi \times 1403{,}57.76$	$Q_2 = 112.27778$
$\sigma_3 = 2\pi \times 1101.7864$	$\omega_3 = 2\pi \times 141{,}195.28$	$Q_3 = 64.075615$
$\sigma_4 = 2\pi \times 1111.3540$	$\omega_4 = 2\pi \times 142{,}400.04$	$Q_4 = 64.066012$
$\sigma_5 = 2\pi \times 637.76409$	$\omega_5 = 2\pi \times 143{,}208.59$	$Q_5 = 112.27395$
$\sigma_6 = 2\pi \times 184.99487$	$\omega_6 = 2\pi \times 143{,}496.83$	$Q_6 = 387.84002$

FIG. 6 POLE-ZERO CONFIGURATION OF THE N3 CHANNEL BAND FILTER CHARACTERISTIC USING EQUIRIPPLE IN-BAND AND OUT-BAND APPROXIMATION.

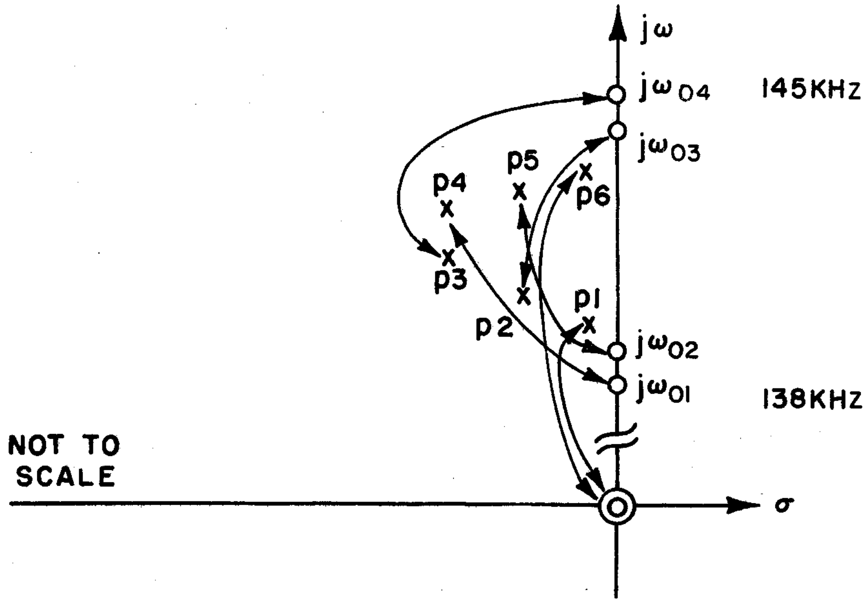


FIG 7 OPTIMUM DECOMPOSITION OF G(s) BASED ON MULTIPARAMETER SENSITIVITY

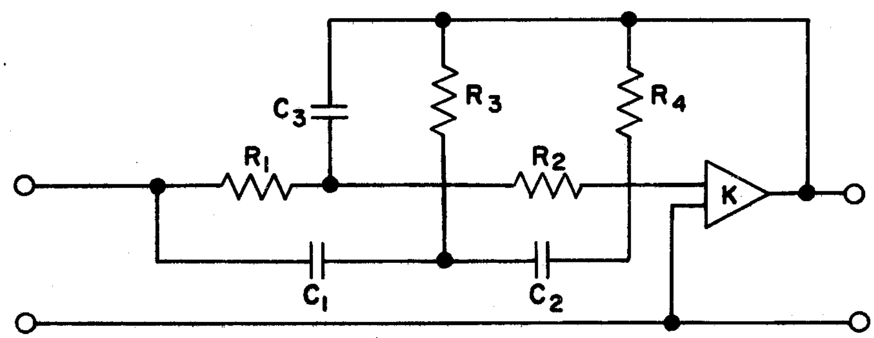


FIG 8 A CANONICAL REALIZATION OF (21)

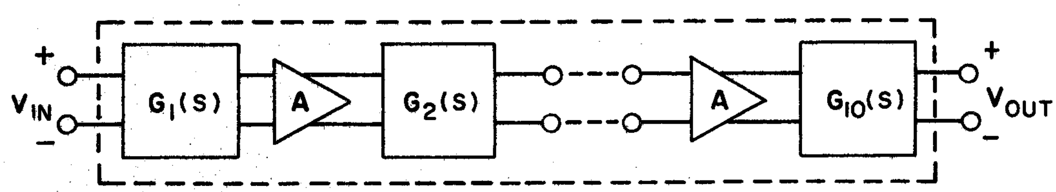
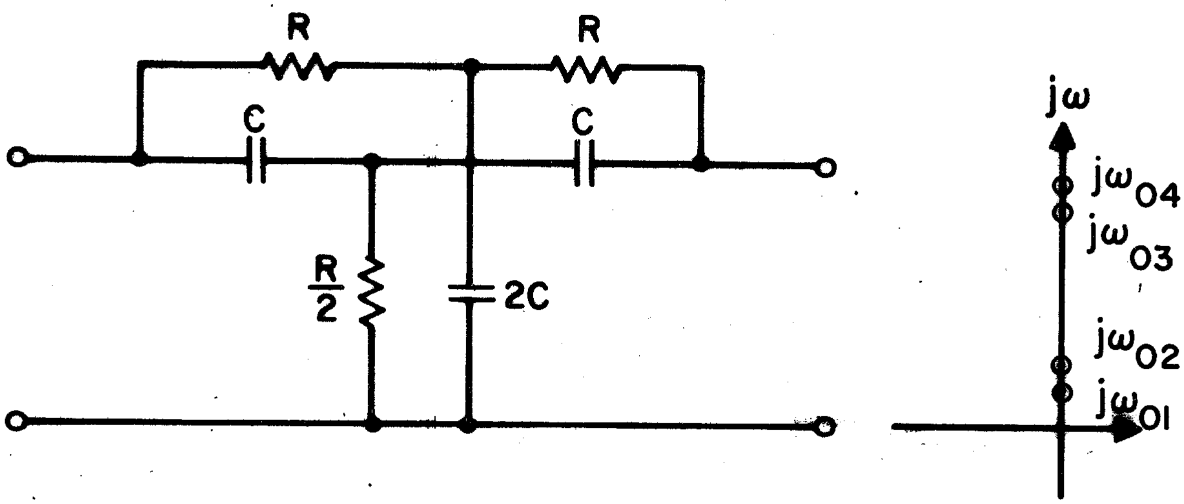


FIG 9 CASCADE SYNTHESIS OF G(s) OF (23)



1. RC-PRODUCT FOR REALIZING ω_{01} IS $RC = \dfrac{1}{2\pi \times 138{,}446.43}$

   $R = 1149.57\,\Omega \qquad C = 1000\,PF$

2. RC-PRODUCT FOR REALIZING ω_{02} IS $RC = \dfrac{1}{2\pi \times 139{,}280.13}$

   $R = 1142.69\,\Omega \qquad C = 1000\,PF$

3. RC-PRODUCT FOR REALIZING ω_{03} IS $RC = \dfrac{1}{2\pi \times 144{,}296.73}$

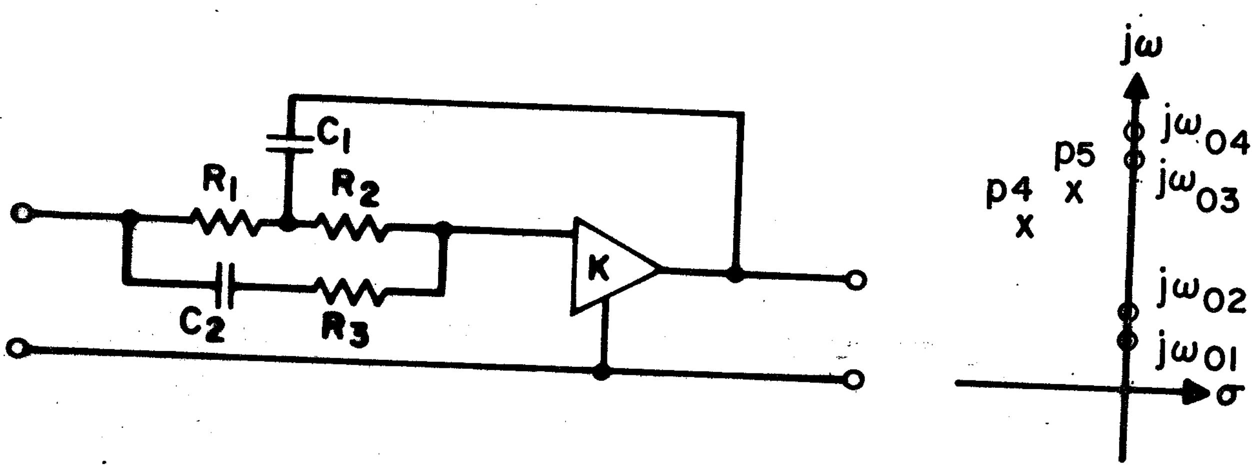
   $R = 1102.96\,\Omega \qquad C = 1000\,PF$

4. RC-PRODUCT FOR REALIZING ω_{04} IS $RC = \dfrac{1}{2\pi \times 145{,}165.72}$

   $R = 1096.36\,\Omega \qquad C = 1000\,PF$

$R = 1000\,\Omega \qquad C \cong 1000\,PF$

FIG. 10 TWIN-TEE REALIZATIONS



1. ELEMENT VALUES FOR THE NETWORK REALIZING p4 :

$K = 8.942$

$R_1 = R_2 = 14.489\ K\Omega$

$C_1 = 39.8\ PF$

$R_3 = 100\ \Omega$

$C_2 = 158.135\ PF$

2. ELEMENT VALUES FOR THE NETWORK REALIZING p5 :

$K = 8.9556$

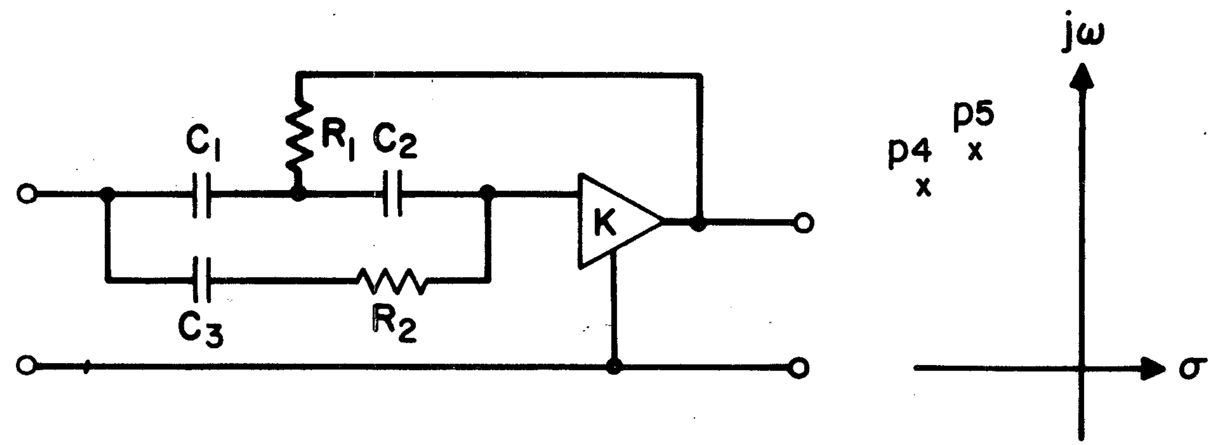
$R_1 = R_2 = 14.697\ K\Omega$

$C_1 = 39\ PF$

$R_3 = 100\Omega$

$C_2 = 154.97\ PF$

FIG. 11 ACTIVE BRIDGED-TEE REALIZATIONS (AB-T1)



1. ELEMENT VALUES FOR THE NETWORK REALIZING p2:

$K = 8.956$ $\qquad R_2 = 4.88\ K\Omega$

$R_1 = 19.388\ K\Omega$ $\qquad C_3 = 0.01676\ \mu F$

$C_1 = C_2 = 113.39\ PF$

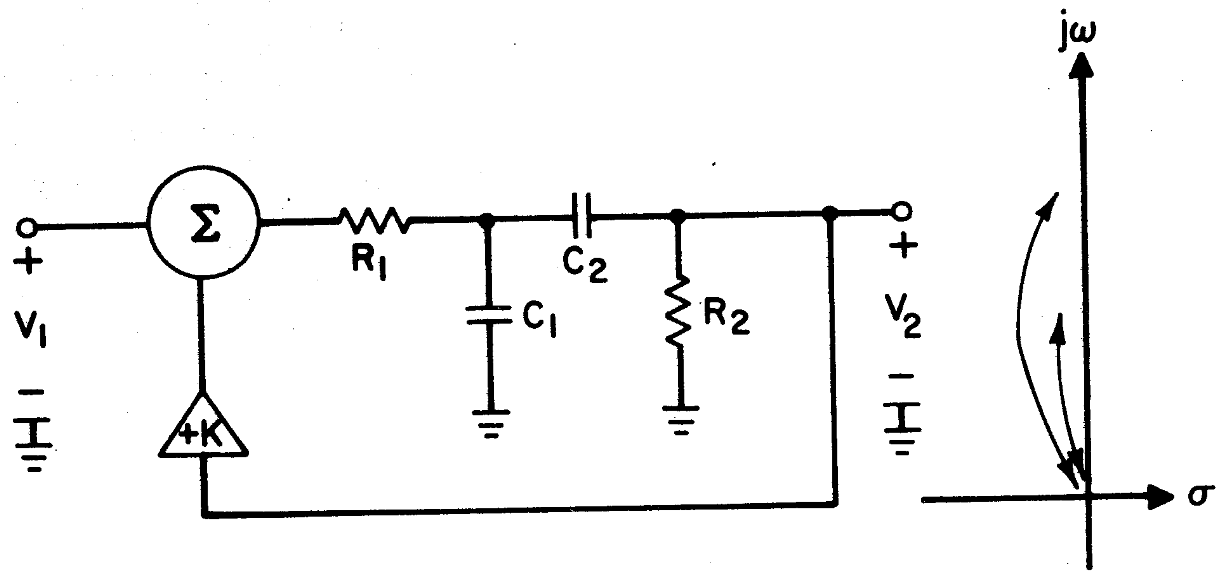
2. ELEMENT VALUES FOR THE NETWORK REALIZING p3:

$K = 8.94$ $\qquad R_2 = 4.88\ K\Omega$

$R_1 = 19.388\ K\Omega$ $\qquad C_3 = 0.0166\ \mu F$

$C_1 = C_2 = 112.716\ PF$

FIG. 12 ACTIVE BRIDGED — TEE NETWORK (AB—T2)



1. ELEMENT VALUES FOR THE NETWORK REALIZING p_1:
   $K = 1.00999$, $R_1 = 100\Omega$ $R_2 = 10K\Omega$ $C_1 = 14.6PF$ $C_2 = 0.08\mu F$
2. ELEMENT VALUES FOR THE NETWORK REALIZING p_6:
   $K = 1.00999$ $R_1 = 100\Omega$ $R_2 = 10K\Omega$ $C_1 = 14.6PF$ $C_2 = 0.08803\mu F$

FIG. 13 HIGH Q ACTIVE FEEDBACK REALIZATIONS (AF)

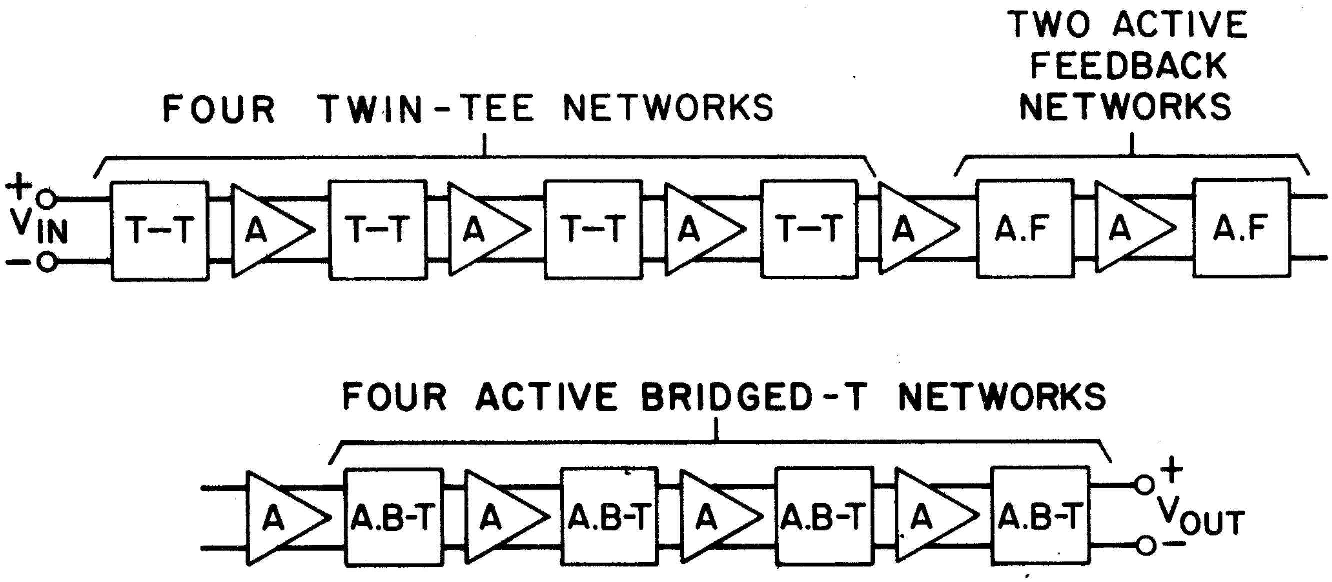


FIG. 14 NETWORK REALIZATION OF N3 CHANNEL BAND FILTER

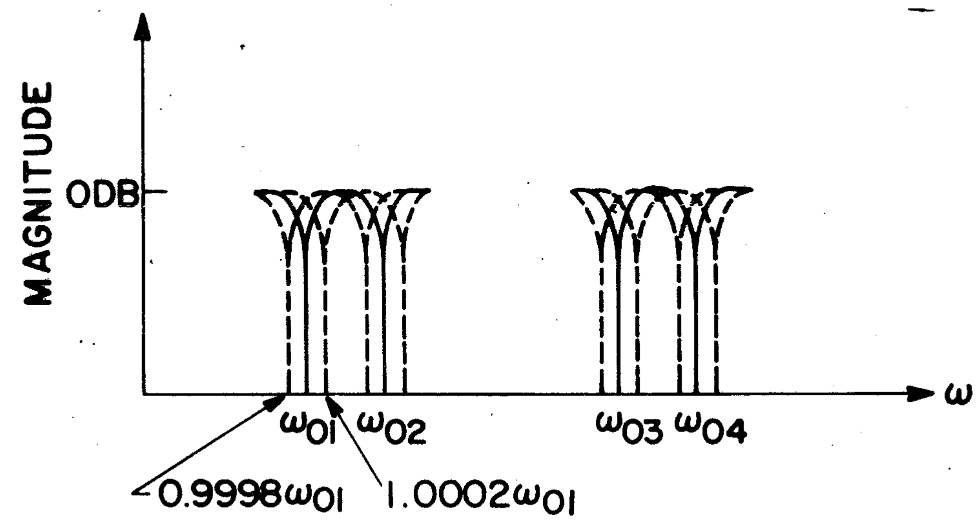


FIG. 15 INITIAL TOLERANCE FUNCTIONAL ADJUSTMENT OF THE FOUR TWIN-TEE NETWORKS

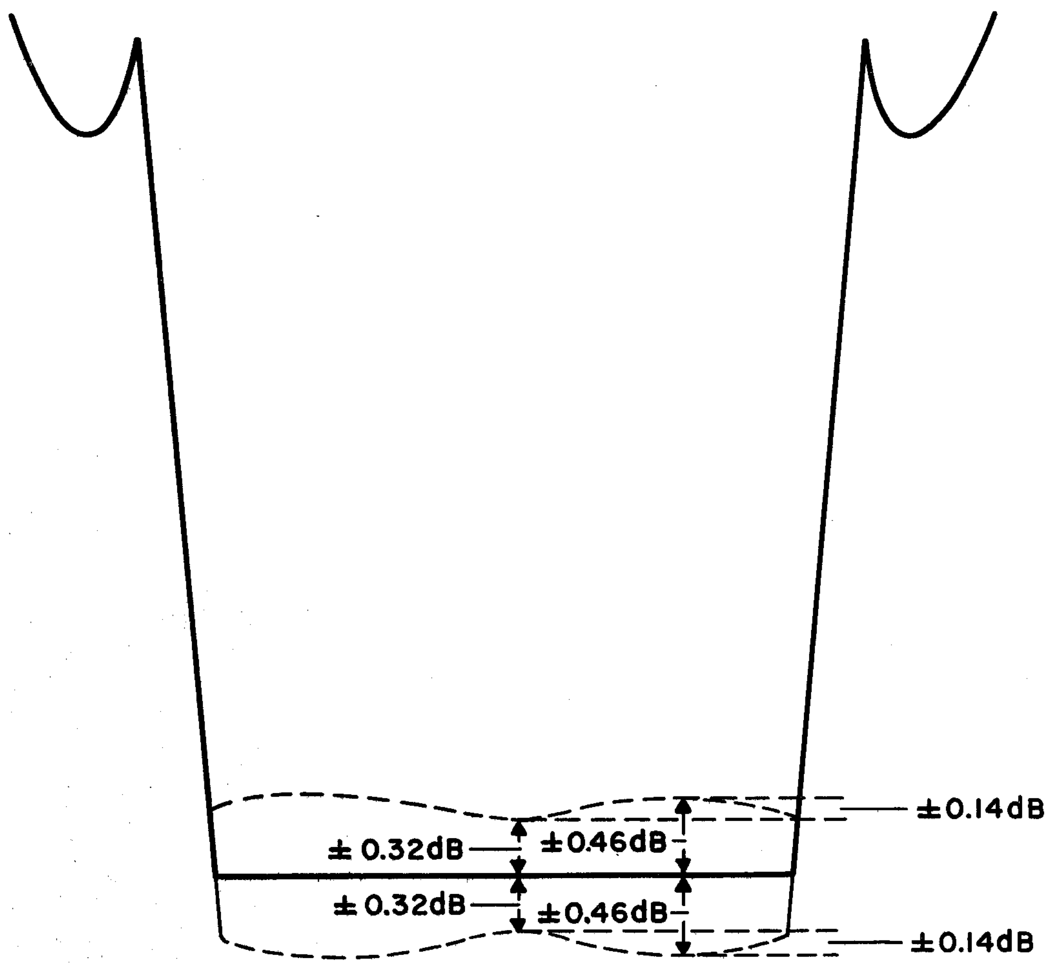


FIG. 16 THE WORST-CASE IN-BAND GAIN DEVIATION DUE TO THE ELEMENT INITIAL TOLERANCES OF THE FOUR TWIN-TEE NETWORKS AFTER FUNCTIONAL ADJUSTMENT.

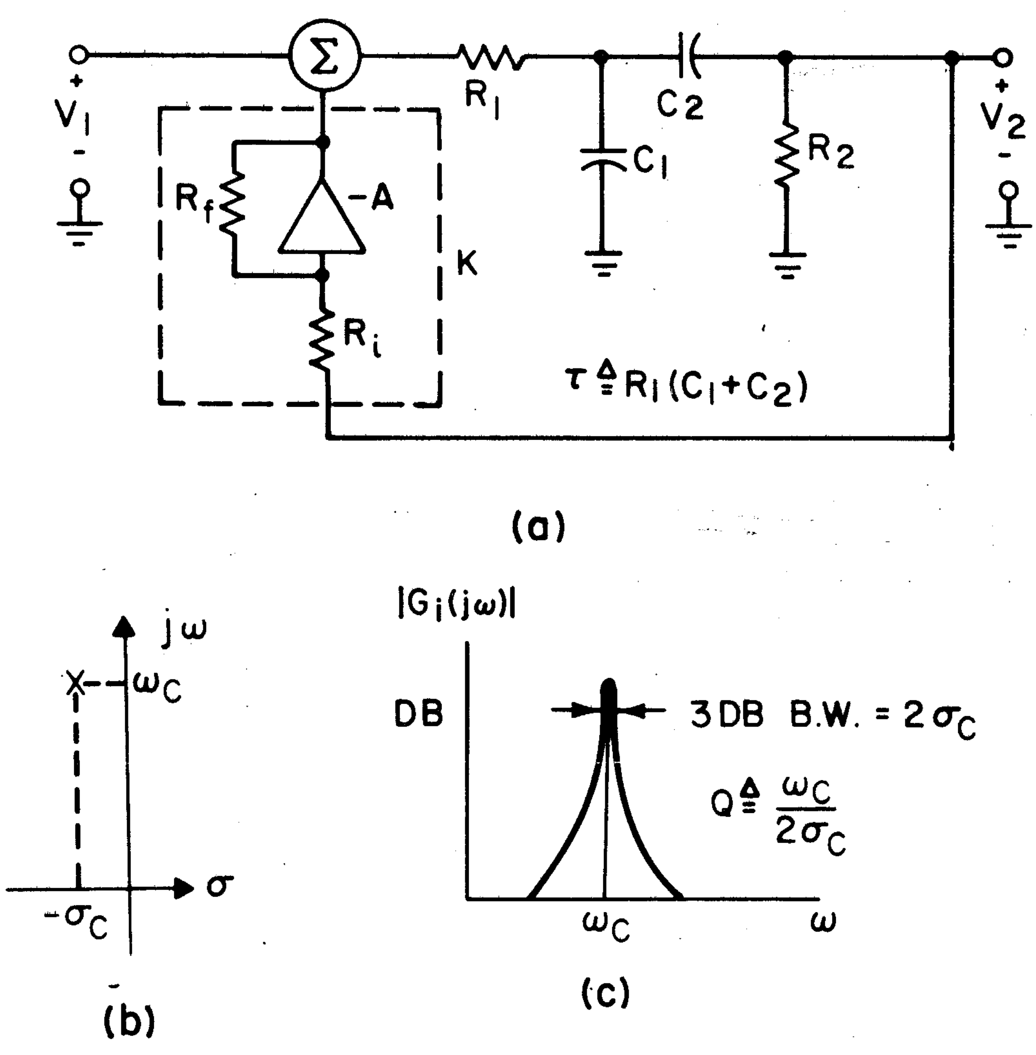


FIG. 17 INITIAL TOLERANCE FUNCTIONAL ADJUSTMENT OF ACTIVE RC NETWORKS.

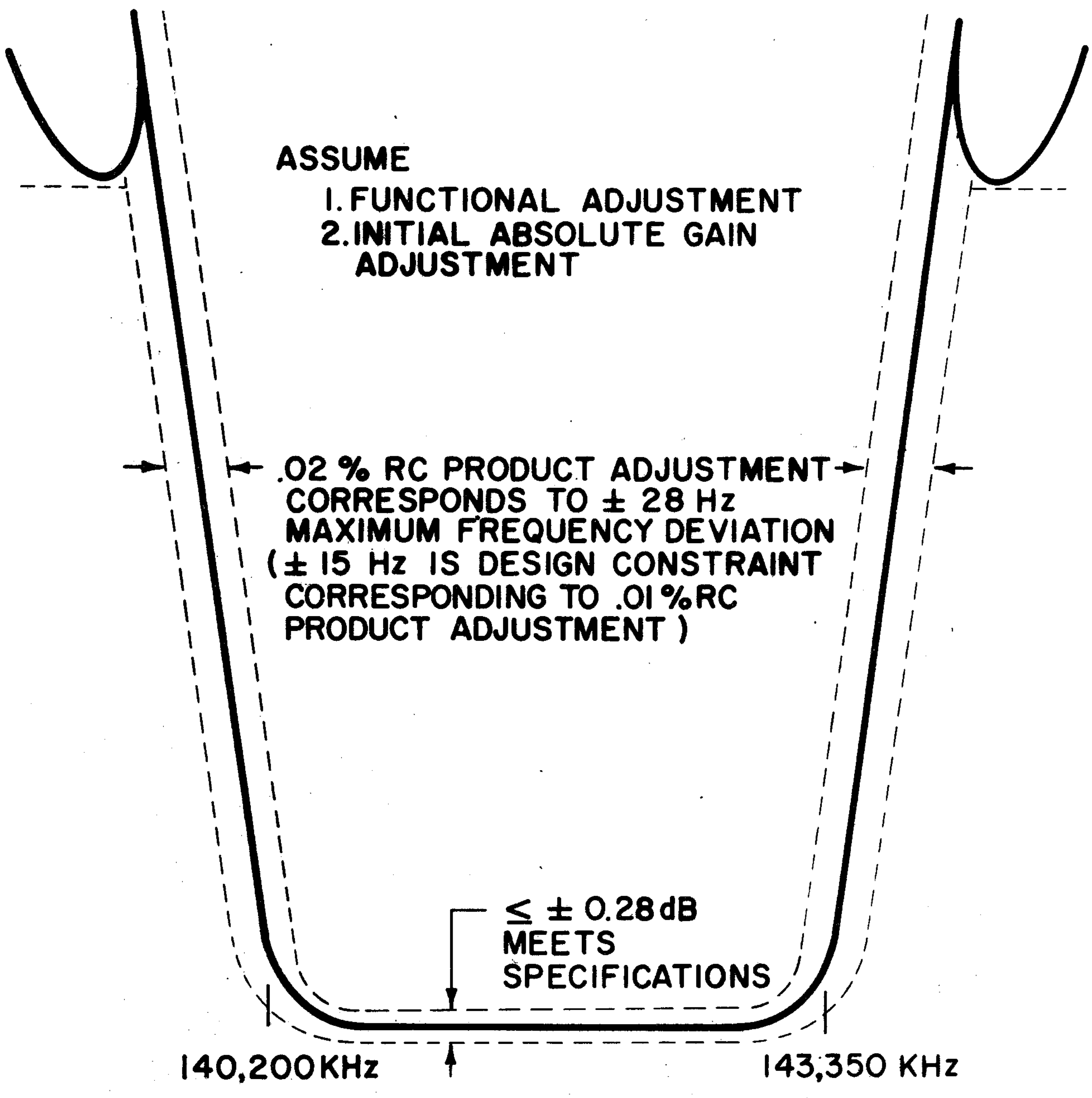


FIG. 18 THE GAIN DEVIATION DUE TO INITIAL TOLERANCE

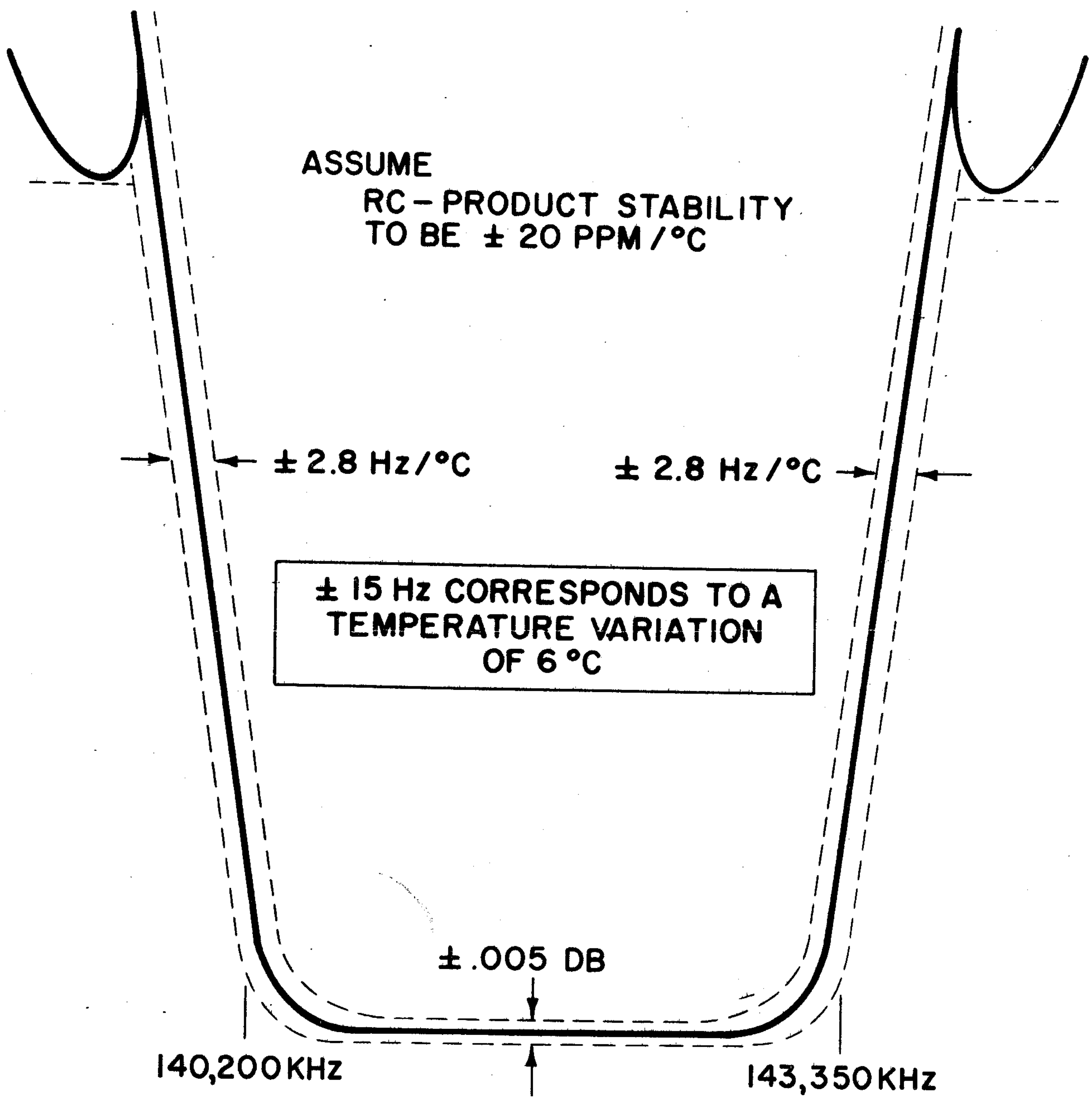


FIG. 19 THE GAIN DEVIATION DUE TO TEMPERATURE TOLERANCE

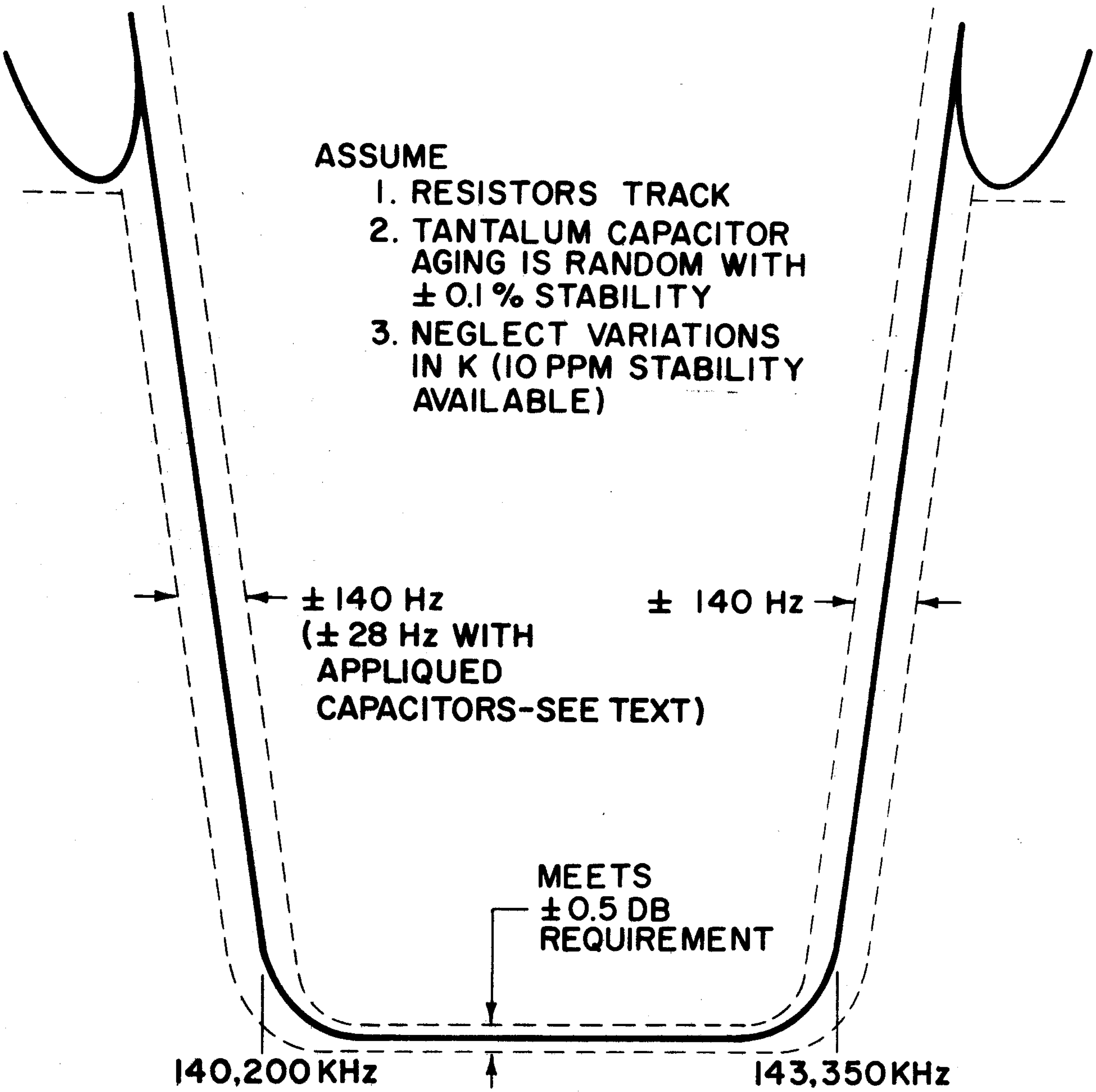


FIG. 20 THE GAIN DEVIATION DUE TO AGING TOLERANCE