COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE— Notes on the Design of Cosine
Roll-Off Filters

MM 67 — 5311 — 18

CASE CHARGED— 38763

FILING CASES— 38763-17

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FILING SUBJECTS— Filter Design
VSB Data Transmission

ABSTRACT

In data transmission systems there is a frequent
need for filters with shaped amplitude response in the pass-
band. In particular, cosine roll-off shaping is commonly
used. A number of filters with various roll-offs have been
designed in connection with the development of several options
of the 203-type Data Set. This memorandum presents the var-
ious filter design and gives an outline of the design pro-
cedure. Also, the relationship between network complexity
and sharpness of roll-off is demonstrated on the basis of
the empirical evidence. It is shown that the complexity of
a 25 percent roll-off filter is approximately 40 percent
greater than that of a 50 percent roll-off filter. Similarly,
a 10 percent roll-off filter is approximately 2.3 times as
complex as a 50 percent roll-off filter. Almost all the
additional complexity results from delay equalization.

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MEMORANDUM FOR FILE

I. Introduction

Raised cosine shaping of the data signal frequency spectrum is commonly used in data transmission systems because of its desirable impulse response properties. Very often it is convenient to do the shaping in the passband rather than in the baseband. A general passband spectrum is shown in Figure 1. The definitions are in terms of a VSB passband spectrum with the carrier frequency located at f_2. In the general VSB case, the high and low end of the spectrum may have different roll-off factors, since one shape provides the equivalent baseband shaping while the other gives the required odd symmetry about the carrier frequency.

The shaping function is frequently equally divided between the transmitter and receiver filters in order to maximize the signal-to-noise ratio in the presence of white noise. Hence, in terms of Figure 1, the ideal filter has the following amplitude characteristic:

$$|H(f)| = \begin{cases} 0 & f < f_A \\[2mm] \left[\dfrac{1}{2} + \dfrac{1}{2}\sin\dfrac{\pi}{2} \times \dfrac{f-f_1}{\alpha_L f_N}\right]^{1/2} & f_A < f < f_B \\[2mm] 1 & f_B < f < f_C \\[2mm] \left[\dfrac{1}{2} - \dfrac{1}{2}\sin\dfrac{\pi}{2} \times \dfrac{f-f_2}{\alpha_H f_N}\right]^{1/2} & f_C < f < f_D \\[2mm] 0 & f_D < f \end{cases}$$

In practice, of course, the ideal shape can only be approximated in the passband and a finite (in db) attenuation can be obtained in the stop bands.

The approximation problem is not an easy one. Although cosine roll-off filters have been designed in the past, the resulting networks sometimes required an unnecessarily large number of components. Recently, however, techniques developed in Department 4734, have been used to produce a number of economical designs. The design method is made possible by the existence of several powerful computer programs - primarily optimization programs. In essence, the method relies on manipulating component values of a bandpass filter to produce a loss characteristic approximating the ideal one. Phase equalization is done separately.

II. Description of Results

Four representative designs are depicted in Figures 2 through 17. For each of the filters, the ideal response (square root shaping), the approximating network and its amplitude and delay response are shown.

The filters, except for Filter 3, are intended for use in various options of the 203-Type Data Set. The requirements for the filters call for the amplitude response to be within +1.0 db of the ideal characteristic over the Nyquist band of frequencies. The actual computed responses, however, are usually within a few tenths of a db over the Nyquist band, with a quite good match over the remainder of the frequency range.

The filters, again with the exception of Filter 3, are delay equalized to have delay distortion within approximately ±50 microseconds over the Nyquist band. In the case of Filter 3, two bridged-tee delay equalizing sections would be necessary to bring the delay distortion within the above tolerance.

Finally, particular to all the designs is the presence of two attenuation peaks, one above and one below the data band. The peaks are located at the frequencies of two pilot tones that are transmitted to facilitate carrier recovery at the receiver. The receiver VSB filter removes

these tones from the data spectrum before demodulation. The ability to control the position of the peaks demonstrates a very useful feature of the design technique.

III. The Design Procedure

As a preliminary step in the design procedure a prototype network is designed that will serve as an initial approximation in the iterative process. In general, the response of this network should be made to resemble the desired one as much as is practical since it may determine whether the iteration will be successful. Unfortunately no guidelines exist that determine what constitutes a sufficiently good approximation; hence, this step is essentially a guess. For the VSB filters, the prototype network is usually a filter with maximally flat passband, 3-db attenuation frequencies the same as those of the final network and out of band loss somewhat in excess of the required loss. The additional loss is a precaution against possible reduction of out of band loss as a result of inband shaping.

The prototype network can be designed through standard techniques. A survey of design methods and available design aids and programs is given in a recent memorandum by G. Szentirmai [1]. For the design of bandpass filters, the Bandpass Filter Design Program [2] is recommended as a very versatile and flexible tool. Theoretical background for the program is given in [3].

Shaping of the loss characteristic of the prototype network is then done by SUPRØX - A General Purpose Successive Approximation Computer Program [4]. This program matches a given function (called a "match function") to an arbitrary curve at specified values of the independent variable. A subroutine VALUE, which is needed to evaluate the match function, usually has to be supplied by the user. A number of special purpose VALUE subroutines exist, among them one that evaluates the insertion loss of a ladder network [5]. Hence, in order to run the program only data cards are needed to specify the requirement curve, frequencies, network topology, element values, and features of program to be used. Among the most useful features are the weighting of requirement points, matching the shape rather than the absolute amplitude (i.e., subtraction of a constant) and, fixing selected parameters before iteration.

When encoding the prototype network, the loss of an inductor can be entered as a series resistance which is kept fixed during iterations. If, after a successful run the inductance differs considerably from its initial value, or if great accuracy is required, the corrected resistance value can be inserted and iteration repeated.

If the prototype network has been designed through the Bandpass Filter Design Program, it usually will have the proper termination on one end only and an impedance transformation will be necessary. It may be advantageous to do this

transformation before using SUPRØX, since great accuracy in computations is not required at this point. Recently a computer program [6] has been made available that does the impedance transformations.

The iteration will not always be successful during the first run. The program may encounter various difficulties or fail to come up with an acceptable solution. Remedies to some of the problems will suggest themselves once familiarity with the program is gained, others will have to be found by trial and error. The last resort is a selection of a new prototype network.

If the SUPRØX is not able to make an improvement at all, other programs might. In Department 4734 good results have been obtained with SIMPLX [7]. Although slower than SUPRØX, SIMPLX has proven to be more reliable in finding a minimum. It is being used to find crude approximation to the desired shape, with SUPRØX refining the match. Particularly useful is the version of SIMPLX written for the CDC 3100 computer, which permits manual interaction with the programs during the approximation. Although the CDC 3100 computer is in the network group area, time on it can be scheduled by outside users.

Once a sufficiently accurate amplitude shaping is obtained, a computation of network response is done to check the effect of shaping at frequencies other than the match points. Phase and delay are also computed to determine the

amount of distortion. Mesh Programs [8] and [9] are well suited to do this, although other equivalent programs exist.

Delay equalization is also done on SUPRØX with the appropriate VALUE subroutine. The number of delay equalizing sections (see Reference [1], Section 3.2) has to be estimated as well as the initial values of parameters for each section. This iteration usually converges easily provided that the initial parameter values are anywhere near the correct value.

Practical delay equalizing sections produce a slight amplitude notch at the frequency of the maximum delay. The depth of the notch will depend on the Q of the inductors and the value of peak delay produced by that section. Hence, a mesh computation of the delay equalized network may be in order as a final check of the network's performance. In extreme cases it may be necessary to predistort the original amplitude requirement for the anticipated effect of delay equalizer on amplitude and repeat the iterative process until satisfactory match is obtained.

IV. Summary

The networks presented here have been designed to satisfy the same delay distortion and stop band attenuation requirements. The bandwidths of the filters are also very similar. Hence, the roll-off factor is the most significant parameter differentiating the networks and some conclusions can be drawn about the effect of roll-off on the complexity

of the networks. This dependence is illustrated in the graph
of Figure 18. The number of inductors in a network is used
as a measure of complexity since at voice frequencies, the
inductor is the largest and most expensive network element.
Included in the data of Figure 18 is a filter with $\alpha_L = \alpha_H =$
0.1, which is being designed in Department 4734. Although
the final design is not available at this writing, the ap-
proximate complexity of the network is known. As might be
expected, sharpening the roll-off increases the filter com-
plexity. The amplitude shaping is accomplished quite easily;
however, delay equalization becomes more difficult. Almost
all of the increase in complexity is due to additional delay
equalization.

V.   Acknowledgment

Acknowledgment is given to Mr. G. Szentirmai for
providing the designs of two filters and his assistance in
the design of the others. The author is also indebted to
Mr. G. Szentirmai for a number of valuable discussions on
design techniques

W. J. Lawless for ARO

HO-5311-ARO-NG   A. R. OLEJNICZAK

Att.
Figures 1-18

References

GENERAL COSINE SHAPED SPECTRUM
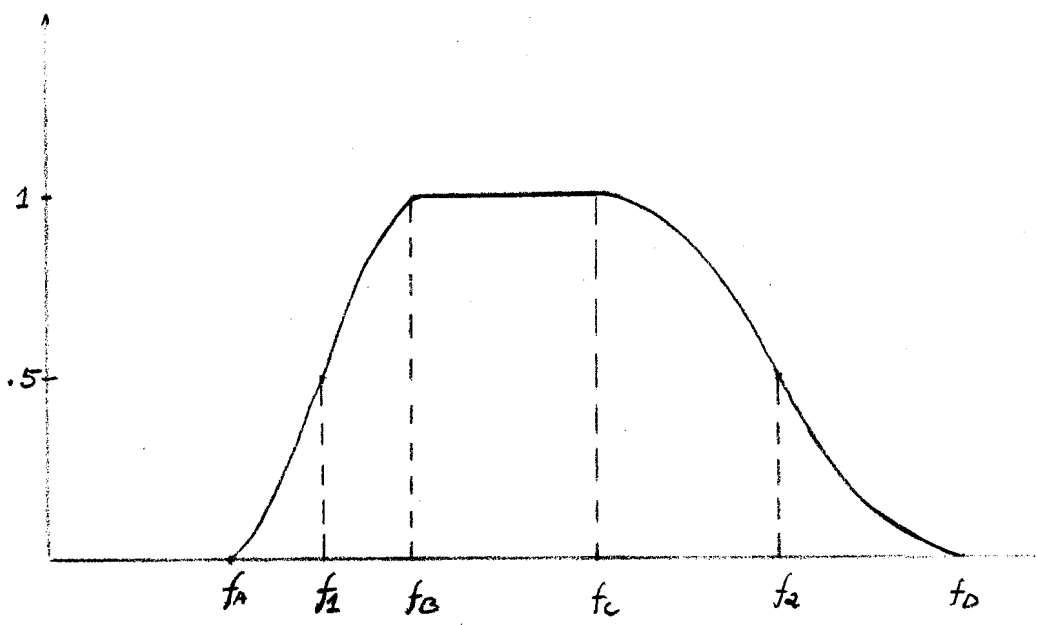
LOW END ROLL-OFF FACTOR:

$$\alpha_L = \frac{f_B - f_1}{f_2 - f_1}$$

HIGH END ROLL-OFF FACTOR:

$$\alpha_H = \frac{f_2 - f_C}{f_2 - f_1}$$

NYQUIST BANDWIDTH:

$$f_N = f_2 - f_1 = \frac{1}{T} \qquad \text{WHERE} \quad \frac{2}{T} \quad \text{IS THE BAUD RATE}$$

FIG. 1

ISSUE

FILTER 1

IDEAL AMPLITUDE CHARACTERISTIC

$$\alpha_L = 0.50$$

$$\alpha_H = 0.50$$

FIG. 2

ISSUE	ENGR ARO	TITLE	BELL TELEPHONE LABORATORIES INCORPORATED	
	DRAWN			SHEET
			NO. OF SHEETS PER SET	

PASS BAND: $1300 \div 2200\,Hz$
TERMINATIONS: $2400\ ohms$.

ELEMENTS:

C_1 = 69.8 nF   570 GH L_1 = 107.8 mH

C_2 = 942. nF   535 HR L_2 = 49.55 mH

C_3 = 165. nF   570 DY L_3 = 335.8 mH   (f=700 Hz)

C_4 = 35.7 nF   570 HB L_4 = 149.8 mH   (f=2700 Hz)

C_5 = 154. nF   570 FT L_5 = 192.1 mH

C_6 = 40.2 nF   570 EN L_6 = 247.4 mH ⎫

C_7 = 11.5 nF   570 HY L_7 = 148.2 mH ⎭ (f=1993.8 Hz)

C_8 = 23.2 nF   570 PF

C_9 = 39.2 nF   570 GU

C_{10} = 15.0 nF   570 FP

C_{11} = 21.5 nF   570 HK

C_{12} = 21.5 nF   570 HK

ALL INDUCTORS 1647 TYPE.

FIG. 3

JUL 15 1967 CODER ARO PROB   RUN   OLEJNICZAK   B, D-B
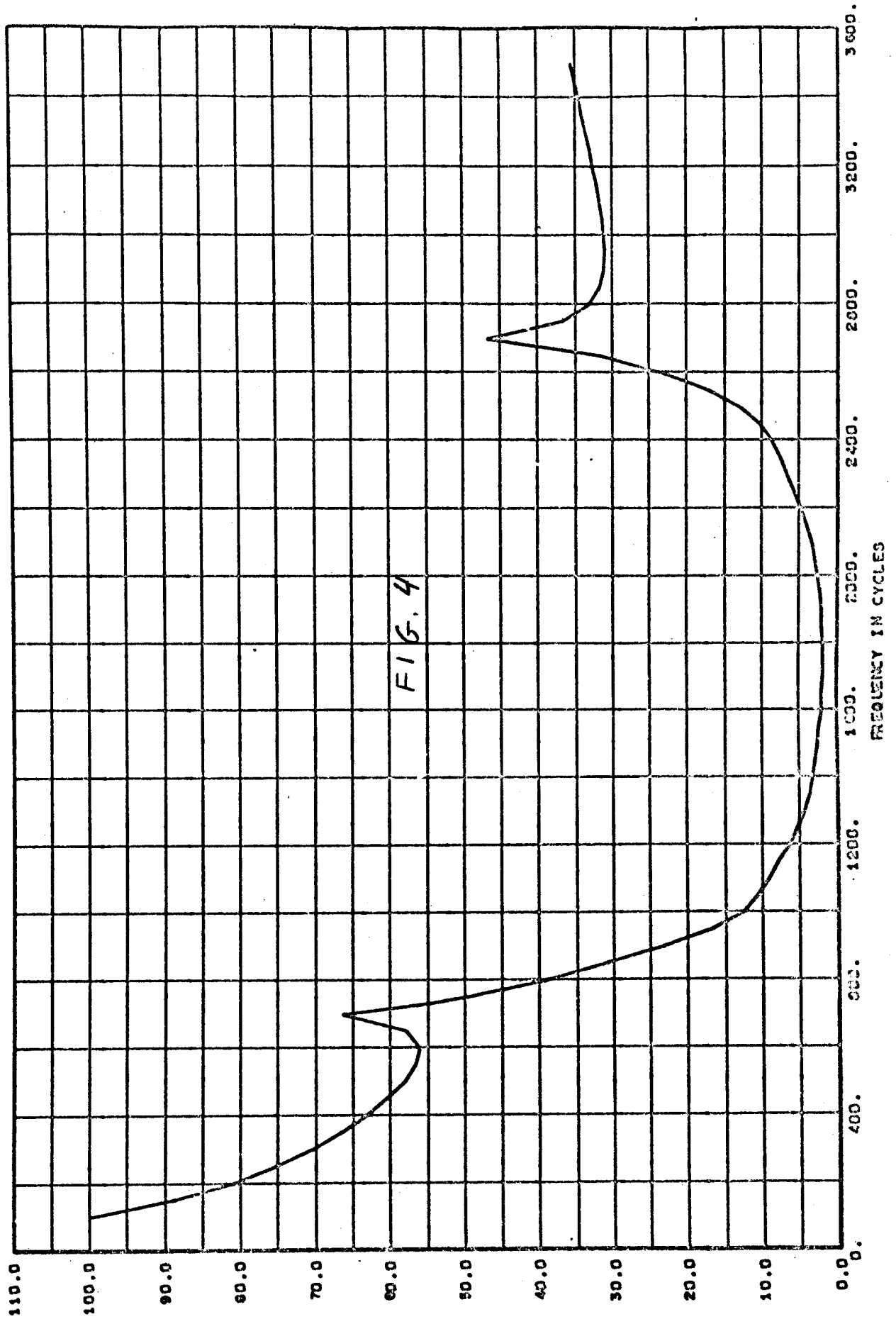
FIG. 4

FREQUENCY IN CYCLES

TRANSMISSION LOSS IN DB
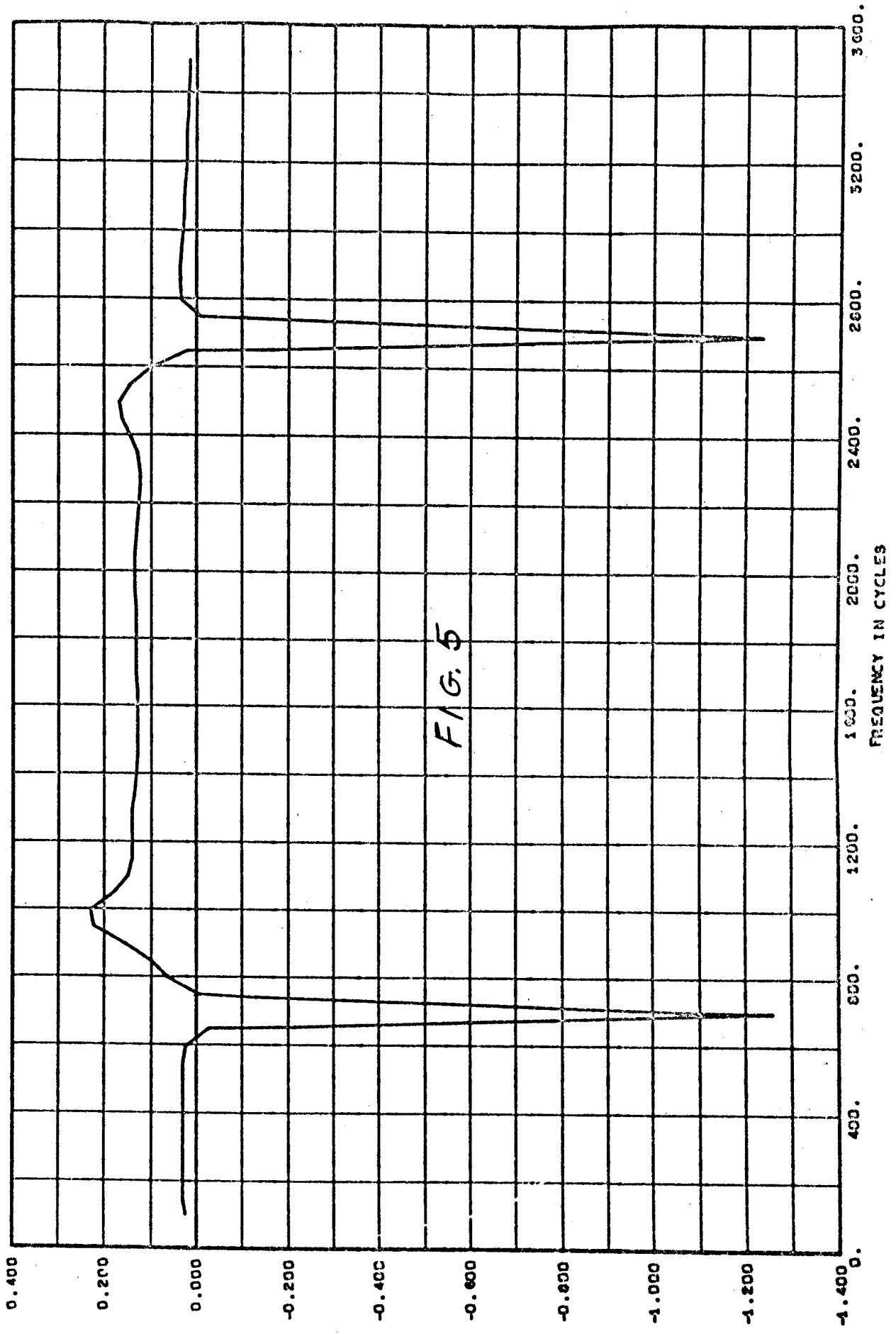
FIG. 5

FILTER 2

IDEAL AMPLITUDE CHARACTERISTIC
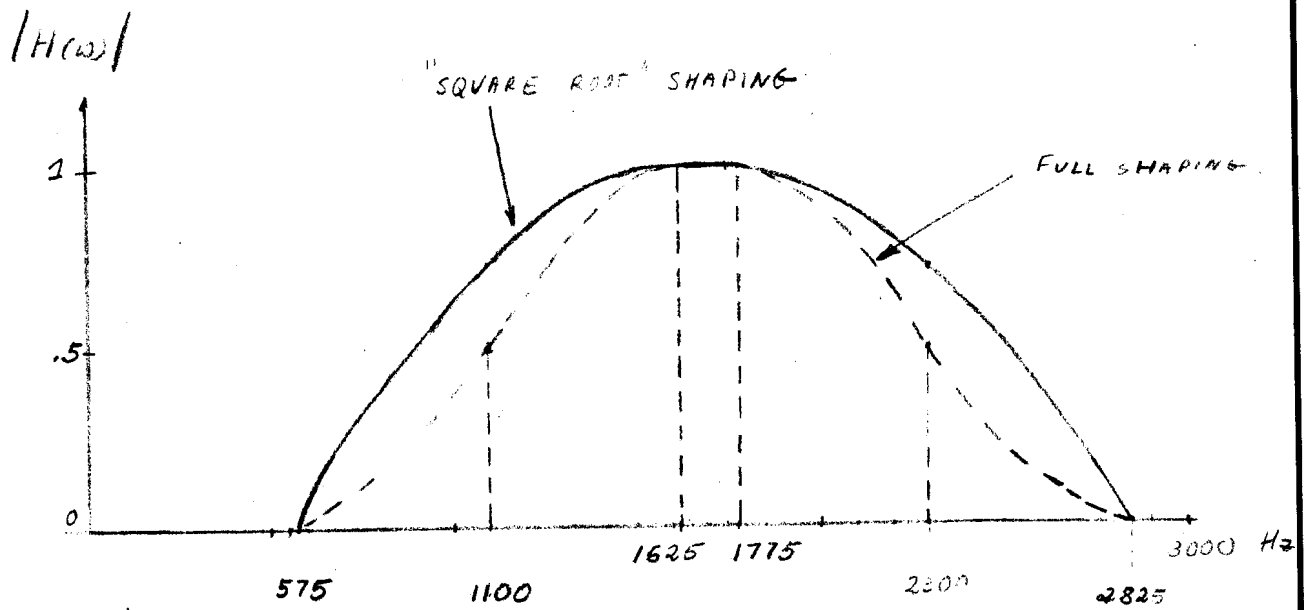
$|H(\omega)|$

"SQUARE ROOT" SHAPING

FULL SHAPING

1

.5

0

575   1100   1625 1775   2300   2825   3000 Hz

$\alpha_L = 0.44$

$\alpha_H = 0.44$

FIG. 6

7 11/16

10 5/8

ISSUE
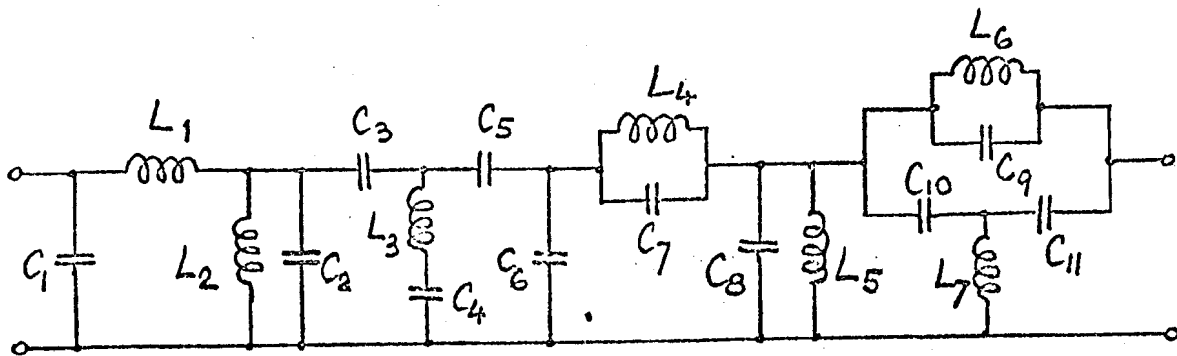
PASS BAND: $1100 \div 2300$ Hz

TERMINATIONS: 2400 ohms

ELEMENTS:

$C_1 =$ 60.4 nF   570 PL $L_1 =$ 99.25 mH

$C_2 =$ 162. nF   570 FS $L_2 =$ 45.21 mH

$C_3 =$ 84.5 nF   570 BR $L_3 =$ 482.5 mH $(f = 500$ Hz$)$

$C_4 =$ 210. nF   570 AJ $L_4 =$ 136.3 mH $(f = 2900$ Hz$)$

$C_5 =$ 73.2 nF   570 FC $L_5 =$ 413.5 mH

$C_6 =$ 8.66 nF   570 JF $L_6 =$ 295.4 mH $\Big\}\,(f = 1981.5$ Hz$)$

$C_7 =$ 22.1 nF   570 LY $L_7 =$ 126.5 mH

$C_8 =$ 33.2 nF   570 HD

$C_9 =$ 9.09 nF   570 PD   ALL INDUCTORS 1647 TYPE.

$C_{10} =$ 25.5 nF   570 CH

$C_{11} =$ 25.5 nF   570 CH

FIG. 7

FIG. 8

REQUENCY IN CYCLES

90.00
80.00
P.
80.00
50.00
40.00
30.00
20.00
10.00
0.00

400.   800.   1200.   100.   2000.   2400.   2800.   3200.   3600.

FIG. 9

JULI 5 19 67 CODER ARO PROB   RUN   OLEJNICZAK   B,D-A

REQUENCY IN CYCLES
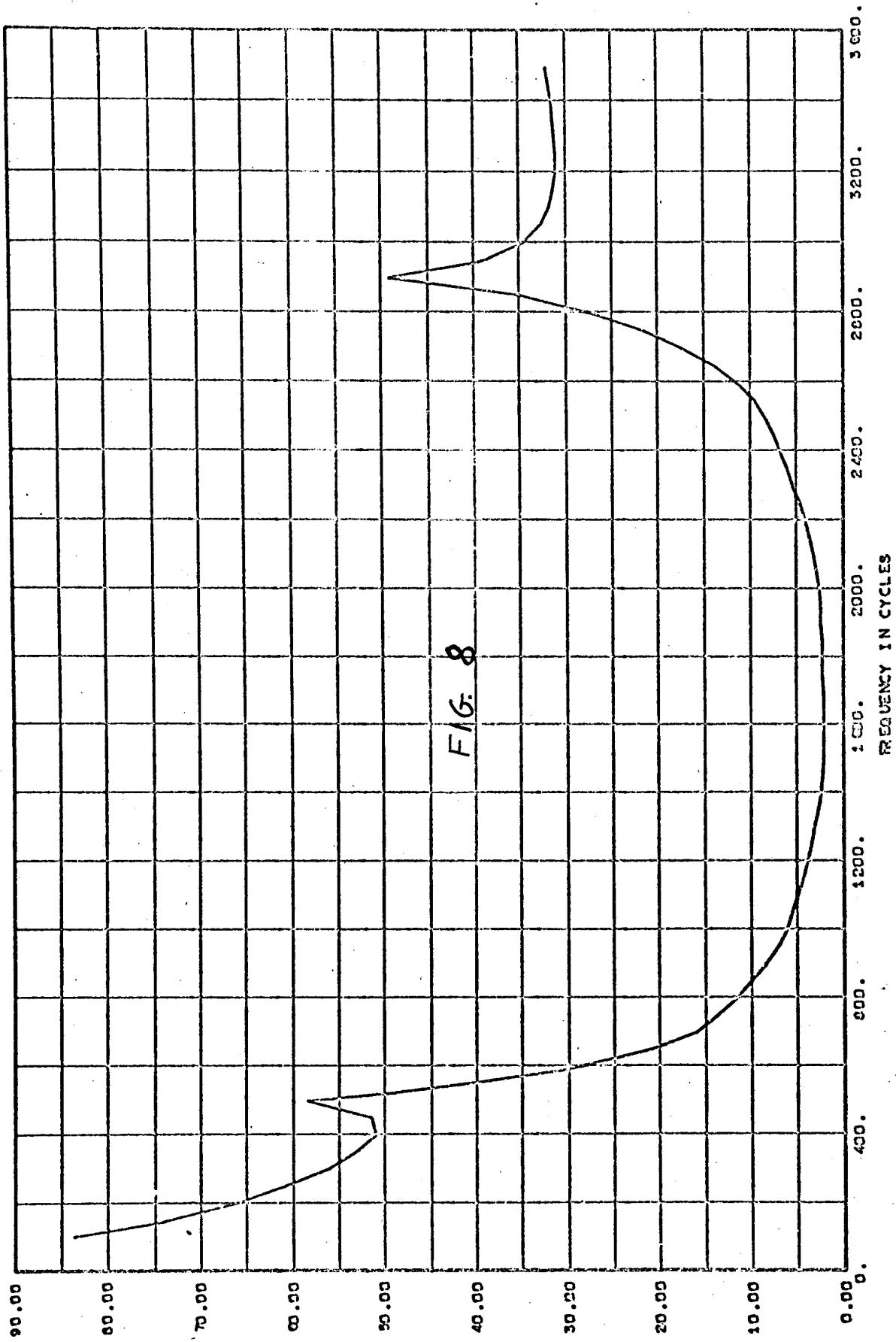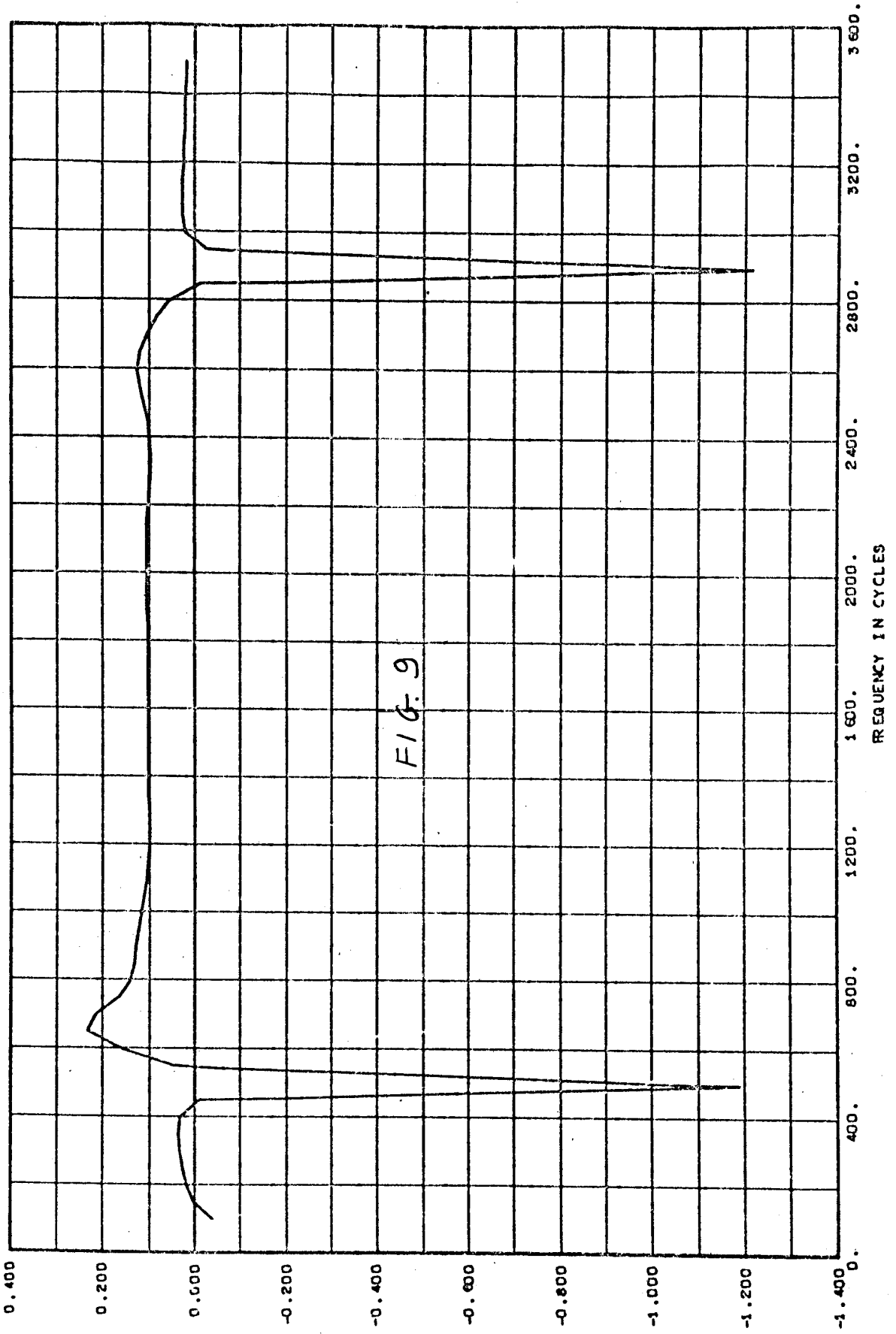
FILTER 3

IDEAL AMPLITUDE CHARACTERISTIC

$|H(f)|$

"SQUARE ROOT" SHAPING

1.0 —

.5 —

0

500   1,000   1500   2000   2500   3000 Hz

750   1100   1450   1950   2300   2650

$\alpha_L = .29$

$\alpha_H = .29$

FIG. 10

ISSUE

7 11/16

10 5/8

$C_1 = 13.85 \ nF$

$C_2 = 45.27 \ nF$

$C_3 = 97.62 \ nF$

$C_4 = 288.2 \ nF$

$C_5 = 84.87 \ nF$

$C_6 = 112.3 \ nF$

$C_7 = 3.85 \ nF$

$C_8 = 34.0 \ nF$

$C_9 = 26.7 \ nF$

$L_1 = 271.83 \ mH$

$L_2 = 35.61 \ mH$

$L_3 = 179.6 \ mH$

$L_4 = 385.1 \ mH$

$L_5 = 732. \ mH$

$L_6 = 325.2 \ mH$

INDUCTORS: 1647·TYPE

CAPACITORS: 570 TYPE

TERMINATIONS: 2400 ohms

FIG. 11
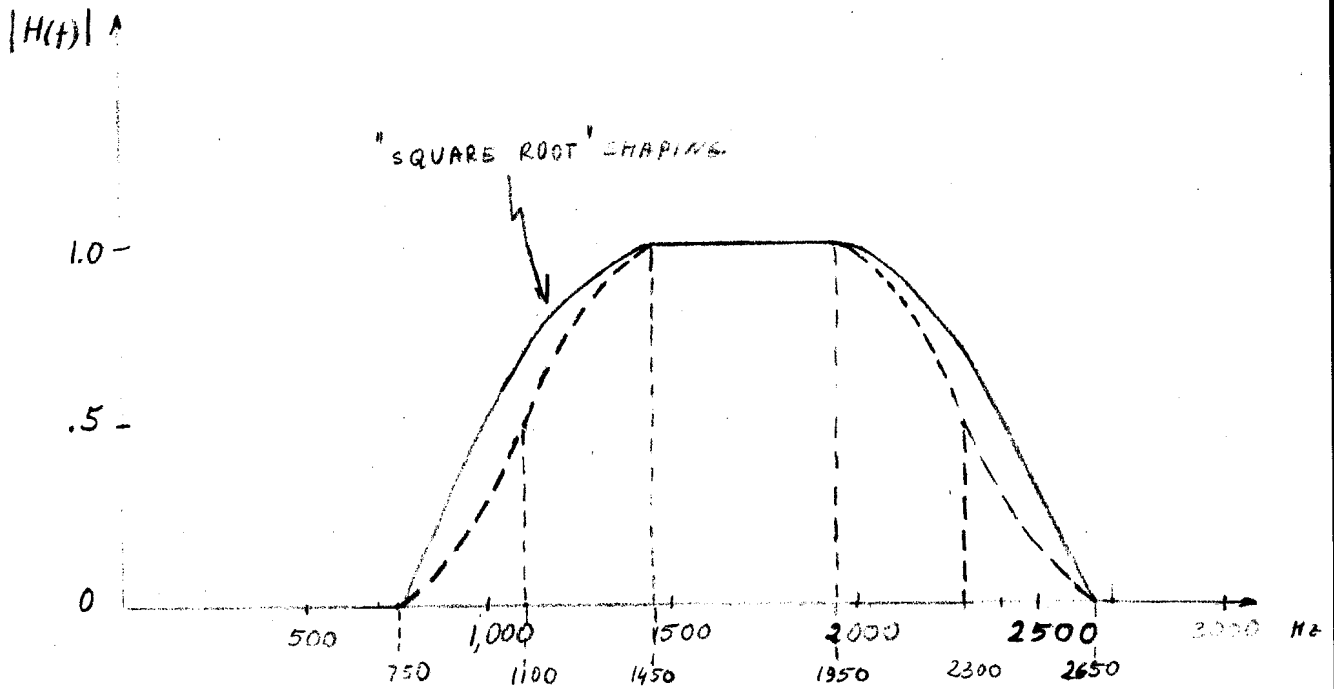
ISSUE	ENGR	TITLE	BELL TELEPHONE LABORATORIES
	ARO		INCORPORATED
	DRAWN		NO. OF SHEETS PER SET / SHEET

JUN20 1967CODER   PROB   RUN   OLEJNICZAK   VSB-C

FIG. 12
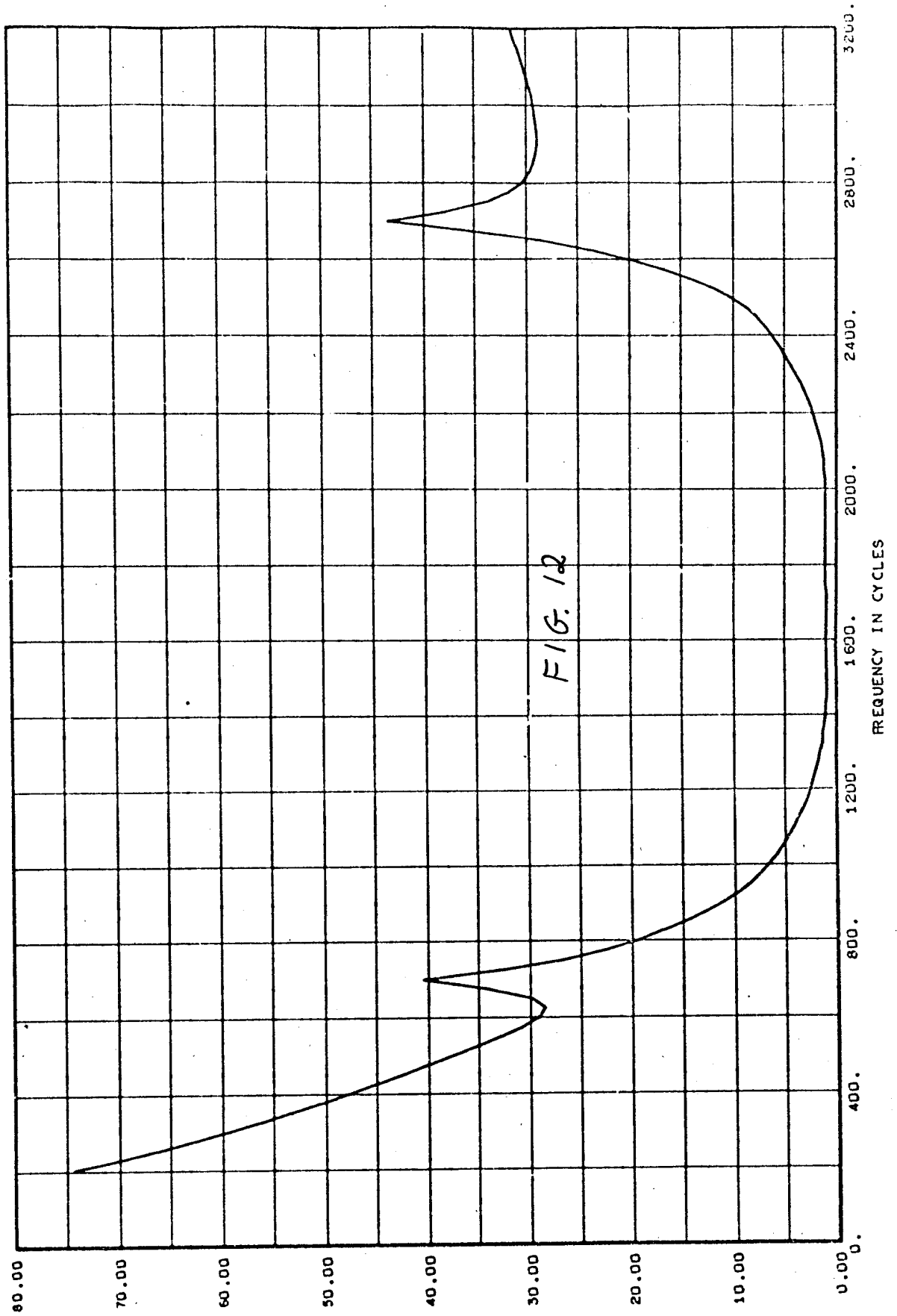
FREQUENCY IN CYCLES

INSERTION LOSS IN DB

FIG. 13

FREQUENCY IN CYCLES
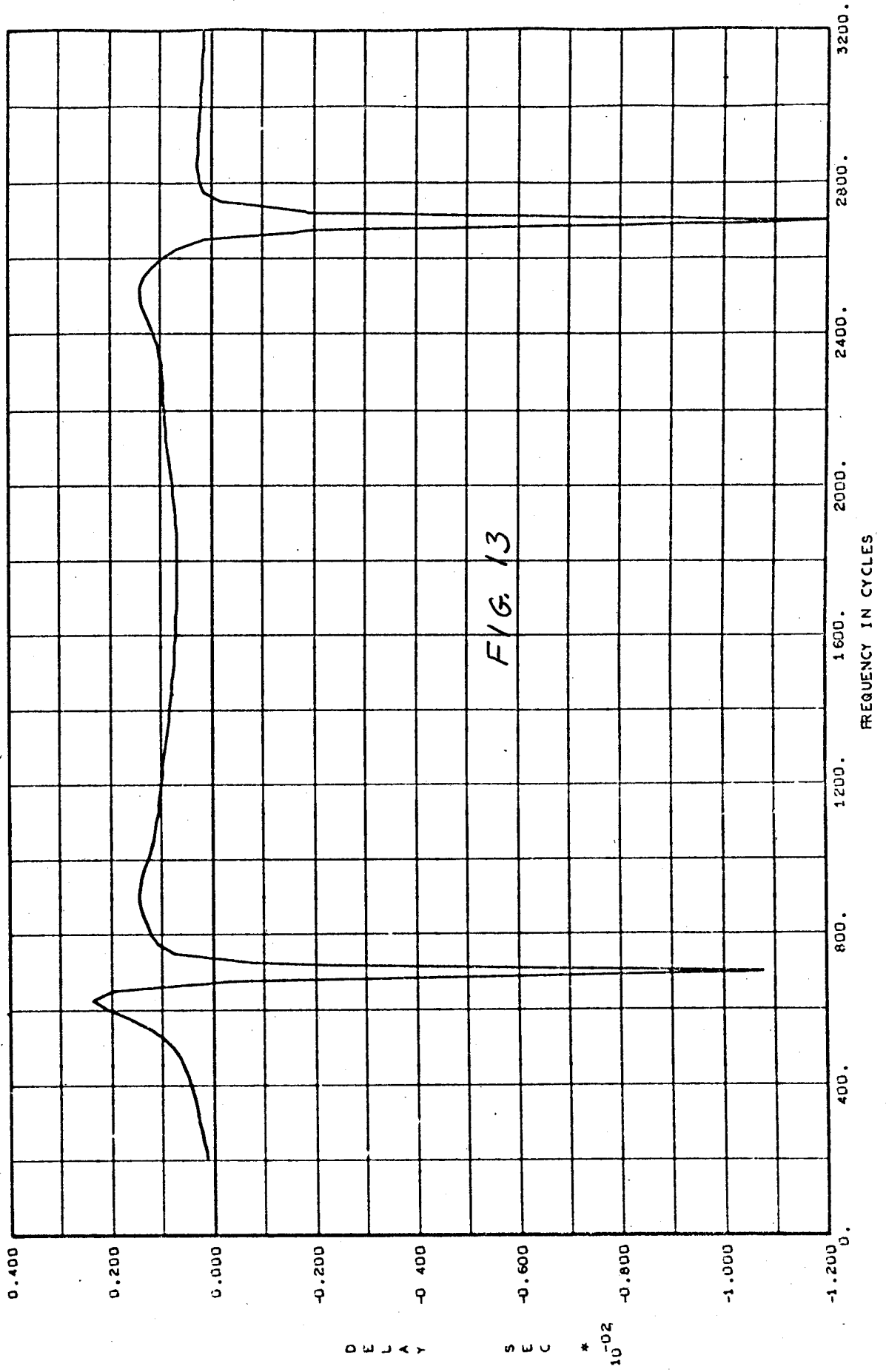
FILTER 4
IDEAL AMPLITUDE CHARACTERISTIC

"SQUARE ROOT" SHAPING

$\alpha_L = 0.22$

$\alpha_H = 0.28$

FIG. 14

ISSUE	ENGR ARO	TITLE	BELL TELEPHONE LABORATORIES INCORPORATED	
	DRAWN 9/5/67		NO. OF SHEETS PER SET	SHEET

PASS BAND: 800 HZ – 2400 HZ
TERMINATIONS: 2400 ohms
ELEMENT VALUES:

C_1 = 14.0 nF   570 EP L_1 = 476.4 mH
C_2 = 32.4 nF   570 HE L_2 = 703.6 mH
C_3 = 4.87 nF   570 NU L_3 = 287.7 mH
C_4 = 127. nF   570 PP L_4 = 565.4 mH   (400 Hz)
C_5 = 59.0 nF   570 BW L_5 = 121.0 mH   (2900 Hz)
C_6 = 280. nF   570 RB L_6 = 334.8 mH
C_7 = 24.9 nF   570 T L_7 = 291.0 mH } 1508.6 Hz
C_8 = 53.6 nF   570 BY L_8 = 218.2 mH }
C_9 = 21.5 nF   570 HK L_9 = 156.5 mH } 2216.0 Hz
C_{10} = 25.5 nF   570 CH L_{10} = 188.3 mH }
C_{11} = 25.5 nF   570 CH
C_{12} = 25.5 nF   570 CH
C_{13} = 26.1 nF   570 LF
C_{14} = 13.7 nF   570 HT
C_{15} = 13.7 nF   570 HT

ALL INDUCTORS 1547 TYPE
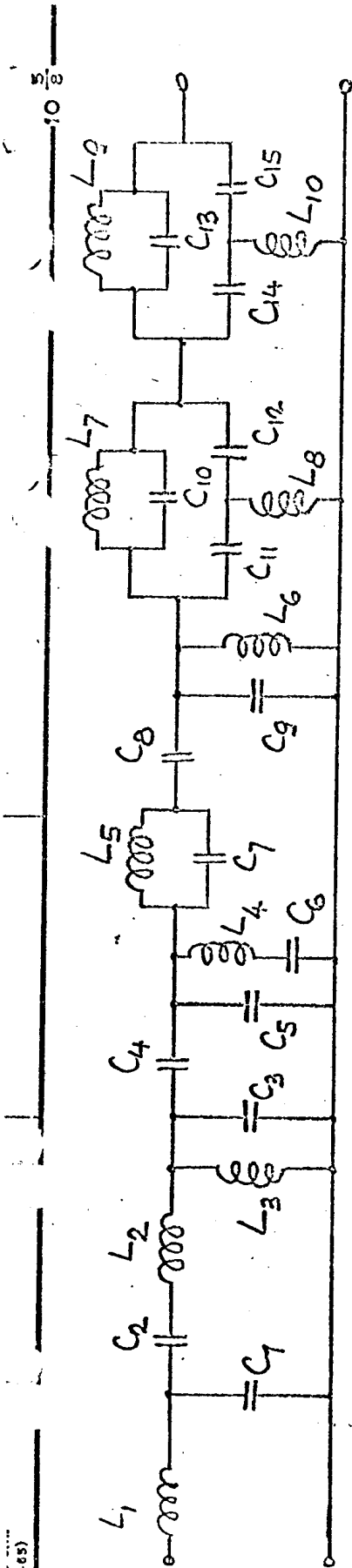
WVH 21020   FIG. 15

ISSUE 1   7.17.67   ENGR ARO   DRAWN   TITLE   VSB TRANSMITTING FILTER   OPTION D

BELL TELEPHONE LABORATORIES INCORPORATED
SHEET
NO. OF SHEETS PER SET

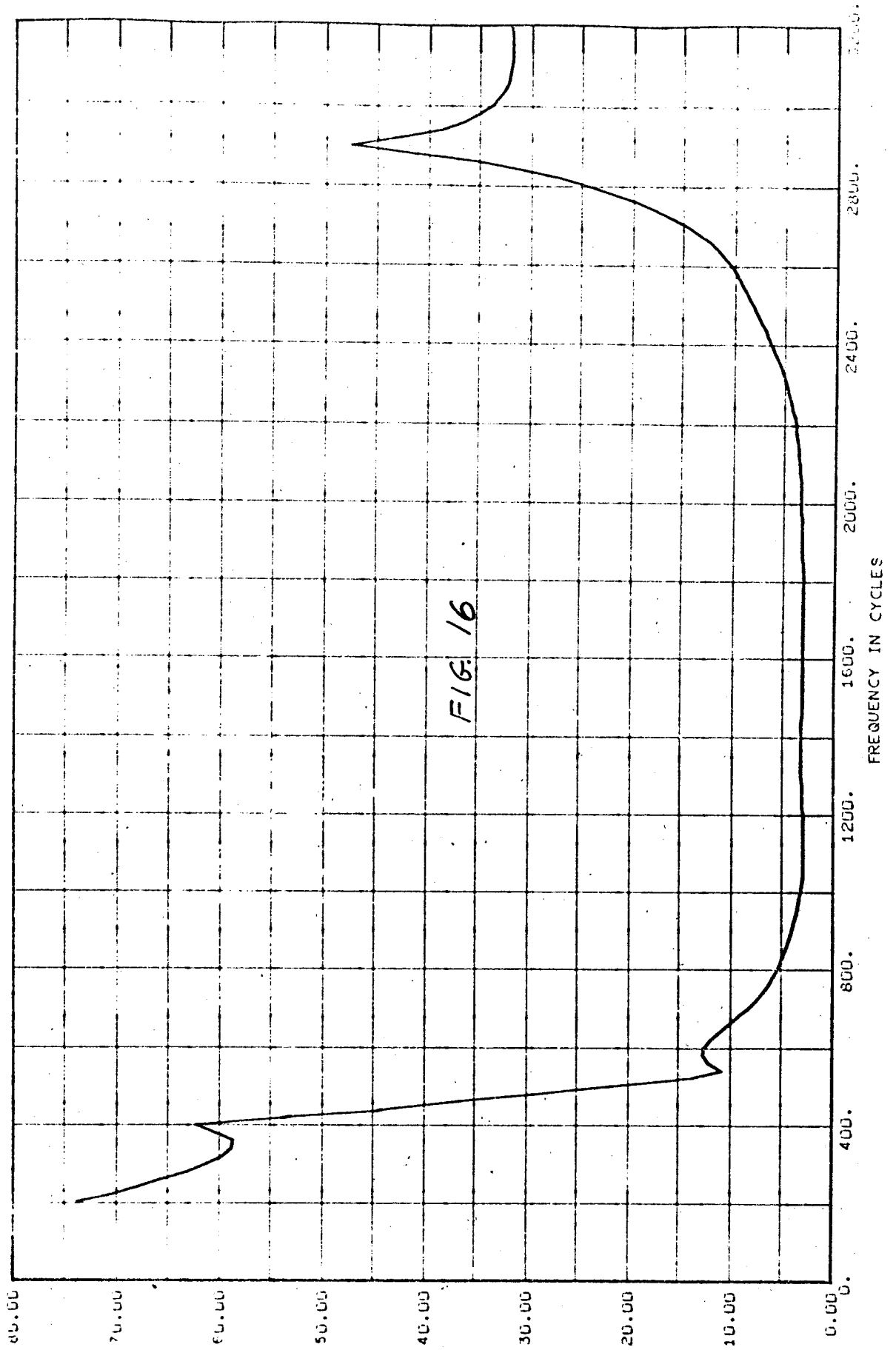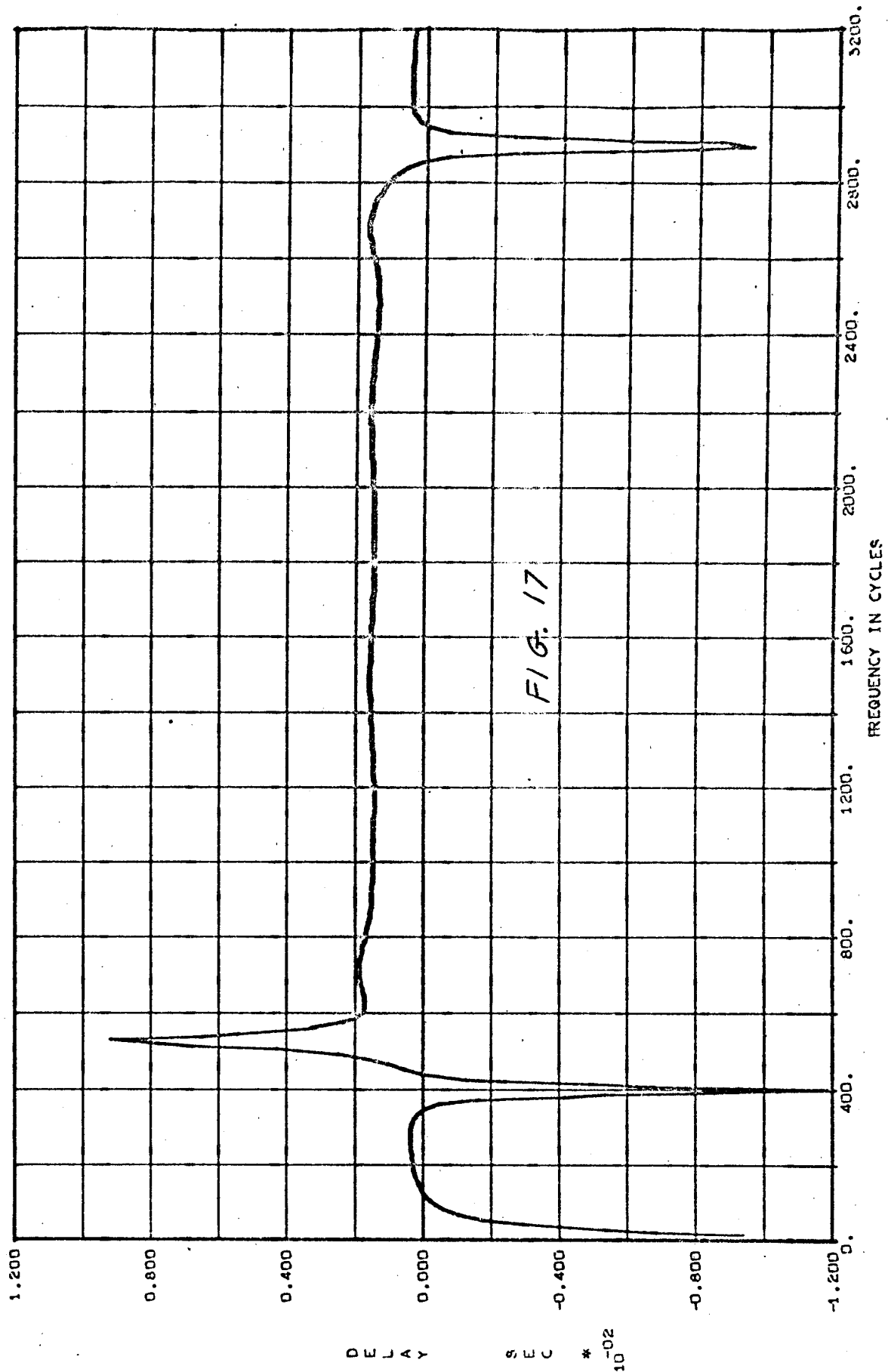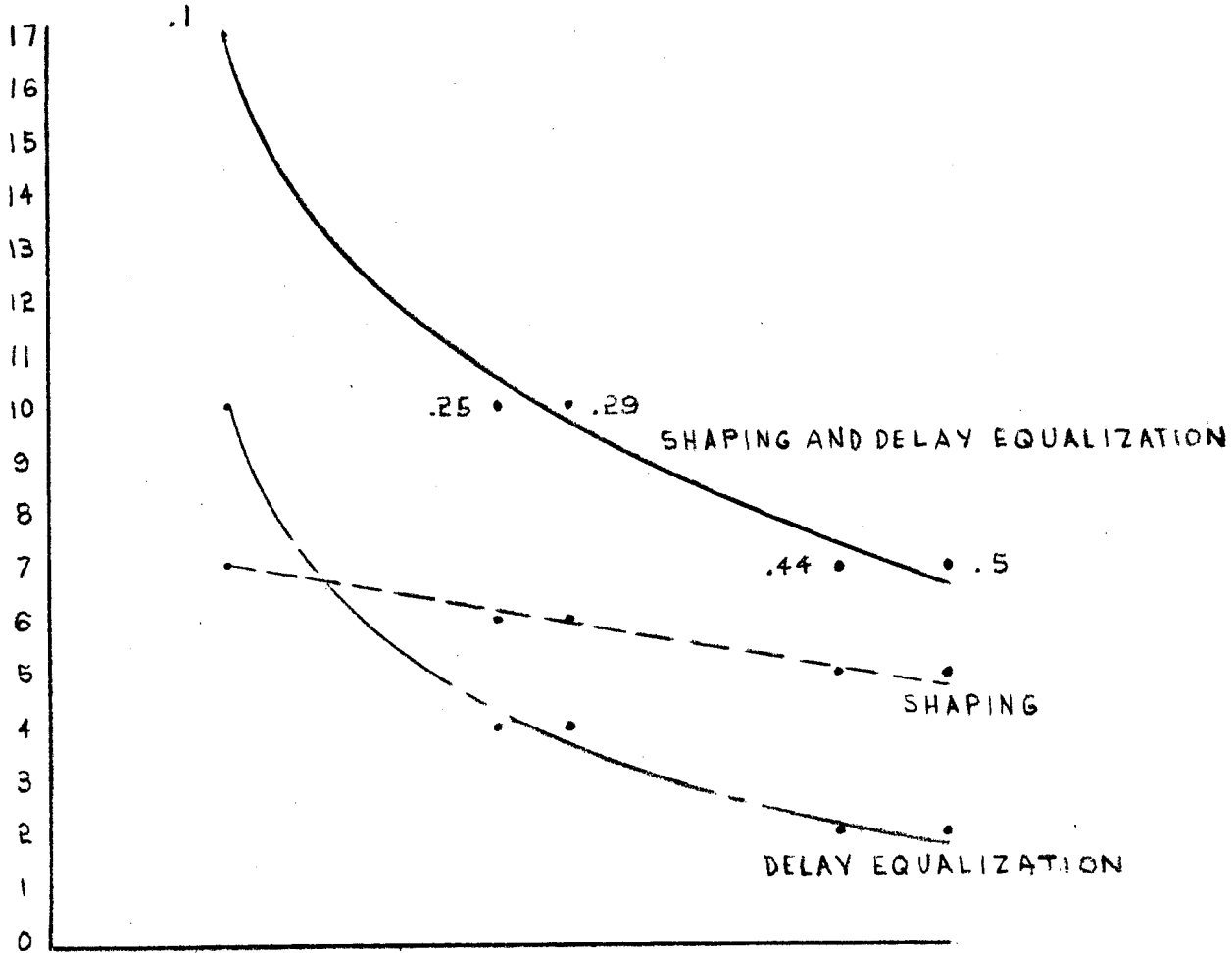MAR5 1967CODER ARO FROB 5123 RUN 1   OLEJNICZAK39027   E,D-C

FIG. 16

FREQUENCY IN CYCLES

MAR5 1967COOÉR ARO PROB 5123 RUN 1   OLEJNICZAK39027   B.D-D

FIG. 17

FREQUENCY IN CYCLES

DELAY   SEC   * 10^{-02}

AVERAGE ROLL OFF = $\dfrac{\alpha_L + \alpha_H}{2}$

FIGURE 18

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