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TITLE— An Electromechanical Filter Below 500 Hz

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CASE CHARGED— 39161-19

FILING CASES— 39161-19

DATE— May 3, 1967

AUTHOR — J. S. Jayson
J. C. Irwin

FILING SUBJECTS— Electromechanical Filters

## ABSTRACT

Electromechanical filters for operation below 500 Hz are considered. Lumped constant parameters are utilized as opposed to distributed parameters to eliminate in-band spurious modes. An asymmetrically excited transducer is used and the effects on the equations of motion are discussed. The equivalent circuit is developed and the theoretical and experimental results compared.

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SUBJECT: An Electromechanical Filter Below 500 Hz - Charging Case 39161-19; Filing Case 39161-19

DATE: May 3, 1967

FROM: J. S. Jayson
J. C. Irwin

MM 67-2655-11

MEMORANDUM FOR FILE

## I. Introduction

It is expected that in future telephone sets the present signaling device, the bell, will be replaced by a tone ringer. One of the requirements of such a system is a filter to pass an audio signal originating at the central office. The selectivity of the filter is to correspond to a Q of 50 to 100. The lower limit is dictated by the desire to discriminate against speech and other telephone signals, while the upper limit is determined by the fact that the incoming signal frequency may vary by $\pm\frac{1}{2}\%$.

We investigate the possibility of using an electromechanical device to meet these requirements. The specifications on selectivity can be met with a simple L-C tank circuit and hence to justify the consideration of an electromechanical filter we must from the outset stipulate that such a device will not require auxiliary coils. Furthermore it should be compact and of simple and economical construction.

An electromechanical filter for frequencies below 20 kHz has been described by Mason and Thurston.[1] This structure is illustrated in Fig. 1a. A major advantage of this structure is the ease of fabrication; it can be stamped from a single piece of metal and the transducers can then be bonded on. Unfortunately, for our application this simple configuration cannot be utilized without some modification. The passband characteristic is derived from a combination of the natural modes of vibration of a bar free at both ends and one clamped at one end, i.e. a cantilever. There is an infinity of such modes. At reasonably high frequencies passbands due to the higher order modes may fall out of the frequency range of interest and if not, they may be suppressed with external circuitry. However, in the present case the frequency of interest is low enough 480 Hz, that the spurious modes fall within the audio range and furthermore we have restricted ourselves to using little or no external circuitry. Therefore to eliminate the spurious modes or drive them out of the audio range, we consider a lumped constant configuration. The structure in Fig. 1a could be altered by weighting the free ends of the bars and the specifications could be met with an input and output transducer without any intervening filter sections. To further simplify the structure it would be desirable if both transducers were located on the same member. Such a filter has been built and is manufactured by the HB Engineering Corporation.* We have built similar filters with both one and two sections.

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*Electronics, Dec. 26, 1966, p. 128.

A single section filter is illustrated in Fig. 1b. The device is supported at or near its nodes by couplers which in this case are thin rods but which could be stamped from the same piece of metal as the web. Piezoelectric elements are bonded to both sides of the web and weights are afixed to the ends.

In the following section we analyze these filters and compare the experimental and theoretical results.

## II. Analysis

### A. Surface of Neutrality

The analysis is similar to that of Reference 1. There are some differences and these will be elaborated upon as we proceed.

The relevant equations for a piezoelectric transducer are:

$$\begin{aligned} &a)\quad E_3 = \frac{D_3}{\varepsilon} - BS_1 \\ &b)\quad T_1 = -BD_3 + Y_D S_1 \end{aligned} \qquad (1)$$

where it has been assumed that $E_3$ is the only component of electric field and $T_1$ is the only stress component. $D_3$ is the electric displacement in the three direction and $S_1$ is the strain in the one direction. The one and three directions refer to the x and z directions respectively. The coordinate system is defined in Fig. 1b. B, $\varepsilon$ and Y are material constants. The longitudinal coupling coefficient squared is given by:

$$k_{13}^2 = \frac{B^2 \varepsilon}{Y_D} \tag{2}$$

To describe the motion of a flexural mode we must obtain an expression for the moment at an arbitrary cross-section. The strain is given by the expression,

$$S_1 = \frac{z - p_n}{R} \tag{3}$$

where $p_n$ represents the coordinate of the surface of neutrality and R is the radius of curvature at the cross-section in question. The surface of neutrality is the surface in a beam undergoing flexure where the stress passes through zero in changing from tensile to compressive. The location of this surface in our problem requires some discussion. If the transducer is symmetrical about the x-y plane, then the surface of neutrality can be determined by symmetry to be located at the center of the web. Our structure is geometrically symmetric, but it is excited asymmetrically in that the input voltage is applied to one piezoelectric element while the output is taken from the other. In the filter of Ref. 1 both the input and output transducers were excited symmetrically.

For a composite beam fabricated from nonpiezoelectric material, the surface of neutrality is easily found by integrating the stress over the cross-section and setting the resulting force equal to zero. The stress is expressed in terms of the strain and the strain is given by Eq. 3. The result is an expression for $p_n$ in terms of geometrical and material constants and independent of the radius of curvature. In a paper by Okamoto et al.,[2] a filter similar to ours was analyzed. The transducer consisted of a free metal bar with a piezoelectric element bonded on one side. The surface of neutrality was located as described above, but this procedure is not strictly correct. The stress in the piezoelectric material is given by Eq. (1) and if this equation is used to determine $p_n$ the result is dependent both upon the radius of curvature, which is dependent on x and upon the voltage across the piezoelectric element. Hence, the surface of neutrality can no longer be described in terms of simple constants. A physical explanation is as follows. Unless we preserve symmetry completely, we excite not only a flexural mode but also a longitudinal mode in the beam. A pure longitudinal mode is characterized by a constant stress at any given cross-section and this stress varies sinusoidally along the length of the beam. If a beam undergoes flexure (see Fig. 2) and an axial force is applied to the beam, the surface of neutrality is shifted. If the axial force should vary from point to point in the x direction and as a function of time, then correspondingly the surface of neutrality would vary. The asymmetric problem thus involves solving for the coupled longitudinal and flexural modes. However, undertaking this task is not necessary if the length of the beam is much greater than the thickness. The flexural mode then occurs at a much lower frequency than the longitudinal mode. The spatial variation due to the longitudinal mode is small in the low frequency region and the two modes are loosely coupled. To solve for the position of the "quasi" neutral surface we write the relation for the longitudinal force,

$$F_{long} = \ell_w \int T_1 \, dz \tag{4}$$

$$= \ell_w \left[ \int_0^{t_1} \left( -BD_{3\,in} + Y_D \frac{(z-p_n)}{R} \right) dz + \int_{t_1}^{t_2} Y_m \frac{(z-p_n)}{R} dz + \int_{t_2}^{t_3} \left( -BD_{3\,out} + Y_D \frac{(z-p_n)}{R} \right) dz \right]$$

where $\ell_w$ = width of beam

$T_1 = Y_m S_1$ is the stress strain relation in the metal.

The net longitudinal force on an element of the beam is given by $\frac{\partial F_{long}}{\partial x} dx$ and this term is normally equated to the inertial force of the element, resulting in the differential equation for the longitudinal mode. To decouple the flexural and longitudinal equation we assume that $\frac{\partial F_{long}}{\partial x}$

is small and set it equal to zero.

$$\frac{\partial F_{long}}{\partial x} = 0 \tag{5}$$

The electrodes on the piezoelectric elements extend along the entire length of the beam and thus,

$$\frac{\partial V}{\partial x} = 0 \tag{6}$$

where V is the voltage across the element. Furthermore for small deflections

$$\frac{1}{R} = - \frac{\partial^2 w}{\partial x^2} \tag{7}$$

where w is the vertical displacement of the beam. After substituting (1a) into (4) to eliminate $D_3$ and using relations (4), (5), (6) and (7) we integrate with respect to z. The result is,

$$\begin{aligned} \frac{\partial F_{long}}{\partial x} = 0 = \ell_w \Bigg[ -\varepsilon B^2\left(p_n t_1 - \frac{t_1^2}{2}\right) + Y_D\left(p_n t_1 - \frac{t_1^2}{2}\right) \\ + Y_m\left(p_n t_m - \frac{t_2^2}{2} + \frac{t_1^2}{2}\right) - \varepsilon B^2\left(p_n t_1 - \frac{t_3^2}{2} + \frac{t_2^2}{2}\right) \\ + Y_D\left(p_n t_1 - \frac{t_3^2}{2} + \frac{t_2^2}{2}\right)\Bigg] \frac{\partial^3 w}{\partial x^3} \end{aligned} \tag{8}$$

Referring to Fig. 1b,

$$t_m = t_2 - t_1$$

$$t_d = t_1 = t_3 - t_2$$

and solving for $p_n$ we find,

$$p_n = t_d + \frac{t_m}{2} \tag{9}$$

which is the result to be expected if longitudinal vibrations are neglected. If we go back to evaluate the inertial term for the longitudinal oscillations we find on inspection that this term is identically equal to zero for the above choice of $p_n$. However, the solution is not self-consistent since we cannot specify the axial force at the ends of the beam. Matching these forces requires two more arbitrary constants which we have discarded along with the longitudinal vibrations.

B. Flexural Equation and Boundary Conditions

The moment at an arbitrary cross-section is,

$$M = \ell_w \int (z - p_n) T_1 \, dz \tag{10}$$

Since there is only one component of electric displacement,

$$\frac{\partial D_3}{\partial z} = 0 \tag{11}$$

Integrating Eq. 1a with respect to z we obtain,

$$V_{in} = \frac{1}{\varepsilon} D_{3\ in} t_d - \frac{B}{R}\left(\frac{t_d^2}{2} - p_n t_d\right) \tag{12}$$

$$V_{out} = \frac{1}{\varepsilon} D_{3\ out} t_d - \frac{B}{R}\left(\frac{t_3^2}{2} - \frac{t_2^2}{2} - p_n t_1\right)$$

where V is the voltage across the piezoelectric element. Integrating (10) after substituting the relationships given above we obtain,

$$M = (V_{in} - V_{out})H_1 - H_2 \frac{\partial^2 w}{\partial x^2}$$

where (13)

$$H_1 = \ell_w t_m \frac{\varepsilon B}{2} (1+u)$$

$$H_2 = \ell_w t_m^3 Y_D \left[\frac{2}{3} u^3 + u^2 + \frac{u}{2} + \frac{Y_M}{12 Y_D} - k_{13}^2 \left(\frac{u^3}{4} - \frac{u}{4}\right)\right]$$

$$u = \frac{t_d}{t_m}$$

The shear force is given by,

$$F_{shear} = \frac{\partial M}{\partial x} \tag{14}$$

and the net force in the z direction on an infinitesimal element is

$$\frac{\partial F_{shear}}{\partial x} dx = \frac{\partial^2 M}{\partial x^2} dx = -H_2 \frac{\partial^4 w}{\partial x^4} dx \tag{15}$$

The inertial force on the same element is, $t_m \ell_w (2\rho_d u + \rho_m) \frac{\partial^2 w}{\partial t^2} dx$. Assuming a harmonic time variation,

$$w = W(x) e^{j\omega t}$$

and equating the two,

$$\frac{\partial^4 W(x)}{\partial x^4} - \beta^4 W(x) = 0$$

$$\beta^4 = \frac{\omega^2 t_m \ell_w (2\rho_d u + \rho_m)}{H_2} \tag{16}$$

The solution to this equation is,

$$W(x) = a_1 \cosh(\beta x) + a_2 \sinh(\beta x) + a_3 \cos(\beta x) + a_4 \sin(\beta x) \tag{17}$$

To evaluate the constants, four boundary conditions must be satisfied. These are the values of the moment and shear force on either end of the bar. For a bar free at both ends these values would be zero. For the current example the moment is equated to the rotational inertial torque applied by the weights to the ends of the bar, and the shear force must be equated to the translational inertial force applied by the weights. Since the entire system conserves momentum and most of the mass is concentrated at the ends, we make the simplifying assumption that the nodes are located at the ends of the beam and replace the boundary conditions relating to the shear force with these conditions. The boundary conditions on the moments are expressed in terms of a mechanical impedance,

$$Z_{MO} = -\frac{M_O}{\dot{\theta}_O}$$

$$Z_{M\ell} = \frac{M_\ell}{\dot{\theta}_\ell} \tag{18}$$

The subscripts O and $\ell$ refer to the values at the respective ends of the bar. $\dot{\theta}$ is the angular velocity and is given by $j\omega \frac{\partial w}{\partial x}$. The minus sign is the result of two factors. First, the impedance of the mass at the end of the bar is the ratio of the torque applied to the mass over the angular velocity. The moment applied by the mass has the opposite sign. Second, a positive moment on the right hand end of the bar results in a negative angular velocity thus explaining the minus sign in only one of the above expressions. The other two boundary conditions are,

$$w = 0 \text{ at } x = 0, \ell \tag{19}$$

Equations (18) and (19) will yield the values of the coefficients of Eq. (17).

C. Equivalent Circuit

Upon integrating Eq. (12) the relations for input and output currents are obtained.

$$i_{in} = j\omega \ell_w \int_0^{\ell} D_{3\ in} dx = j\omega \ell_w \left[ \frac{\varepsilon V_{in} \ell}{t_1} + \frac{\varepsilon B}{2} \left( t_d + t_m \right) \left( \frac{\partial w}{\partial x_\ell} - \frac{\partial w}{\partial x_0} \right) \right] \tag{20}$$

defining,

$$C_0 = \frac{\ell_w \ell \varepsilon}{t_d}$$

we obtain

$$\frac{i_{in}}{j\omega C_0} = V_{in} + \frac{t_d B}{2\ell} \left( t_d + t_m \right) \left( \frac{\partial w}{\partial x_\ell} - \frac{\partial w}{\partial x_0} \right) \tag{21}$$

and similarly,

$$\frac{i_{out}}{j\omega C_0} = V_{out} - \frac{t_d B}{2\ell} \left( t_d + t_m \right) \left( \frac{\partial w}{\partial x_\ell} - \frac{\partial w}{\partial x_0} \right) \tag{21a}$$

Evaluating $\left( \frac{\partial w}{\partial x_\ell} - \frac{\partial w}{\partial x_0} \right)$ we find,

$$\left(\frac{\partial w}{\partial x_1} - \frac{\partial w}{\partial x_0}\right) = \frac{\beta\ H_1(V_{in}-V_{out})}{\Delta}\Big[-j2\omega\beta(Z_{MO}+Z_{M\ell})(\cosh\beta\ell\cos\beta\ell - 1)$$

$$- 4\ H_2\beta^2(\sin\beta\ell - \sinh\beta\ell$$

$$+ \sinh\beta\ell\cos\beta\ell - \sin\beta\ell\cosh\beta\ell)\Big] \tag{22}$$

$$\Delta = -2\omega^2 Z_{M\ell}Z_{MO}\beta^2(1 - \cosh\beta\ell\cos\beta\ell) + j2\omega H_2\beta^3(Z_{MO}+Z_{M\ell})$$

$$\times (\cosh\beta\ell\sin\beta\ell - \cos\beta\ell\sinh\beta\ell) + 4H_2^2\beta^4\sinh\beta\ell\sin\beta\ell$$

In the present example,

$$Z_{MO} = j\omega J_0$$

$$Z_{M\ell} = j\omega J_\ell \tag{23}$$

where $J_0$ and $J_\ell$ are the moments of inertia of the masses at the respective ends of the beam. The mode of interest to us is the lowest mode where the beam is primarily a compliance and most of the inertia is due to the weights. In this case $\beta\ell << 1$ and upon expanding the above equations,

$$\left(\frac{\partial w}{\partial x_\ell} - \frac{\partial w}{\partial x_0}\right) = \left(V_{in}-V_{out}\right)H_1 f(\omega) \tag{24}$$

$$f(\omega) = \frac{-\ell\left(\frac{\omega^2}{3}(J_0+J_\ell)\ell - 4H_2\right)}{\left(\omega^4\frac{J_0J_\ell\ell^2}{3} - \omega^2\frac{4}{3}H_2(J_0J_\ell)\ell + 4H_2^2\right)}$$

Referring to Fig. 3a,

$$i_{in} = j\omega C_0 V_{in} + (V_{in} - V_{out})Y$$
$$i_{out} = j\omega C_0 V_{out} - (V_{in} - V_{out})Y \qquad (25)$$

Comparing these expressions with Equations (21) and (24), we see that

$$Y = \frac{j\omega C_0 H_1 t_d t_m (1+u)B}{2\ell} f(\omega) \qquad (26)$$

A circuit with the same functional dependence as Y is shown in Fig. 3b and the complete equivalent circuit is given in Fig. 3c. The values of the components are,

$$L_1 = \frac{4J_0(1+\gamma)\ell}{\varepsilon B^2 t_m^3 (1+u)^2 u \ell_w C_0 (2+\gamma)}$$

$$L_2 = \frac{L_1 \gamma^2}{4(1+\gamma)}$$

$$C_1 = \frac{k_{13}^2 C_0 (1+u)^2 u}{4\left[\frac{2}{3}u^3 + u^2 + \frac{u}{2} + \frac{Y_M}{12Y_D} - k_{13}^2\left(\frac{u^3}{4} - \frac{u}{4}\right)\right]} \qquad (27)$$

$$C_2 = \frac{C_1(2+\gamma)^2}{3\gamma^2}$$

The substitution $J_\ell = (1+\gamma)J_0$ was performed in deriving the above relations. $R_1$ and $R_2$ are included in the equivalent circuit as a means of representing losses which were neglected in the analysis.

D. Asymmetric Weighting

The parameter, $\gamma$, in Eq. (27) is a measure of the asymmetry of the filter. If the filter is balanced with $J_0 = J_\ell$, $\gamma = 0$, then the parallel resonant circuit in the equivalent circuit is eliminated, $L_2$ going to zero and $C_2$ going to infinity. Note that the resonant frequency of the parallel tank circuit approaches $\sqrt{3}\,\omega_{R1}$ for small $\gamma$ where $\omega_{R1}$ is the resonant frequency of the series circuit. The explanation of these effects is as follows. Referring to Fig. 4a we observe the filter vibrating in its fundamental mode. The bar provides the stiffness of the system and the weights the inertia. Higher order modes of the bar would require that the bar provide both mass and stiffness and these modes are much higher in frequency, except for the mode illustrated in Fig. 4b. Here again the beam need provide only stiffness to the system.* In this mode the beam is stiffer and hence the higher resonant frequency represented by the parallel resonant circuit. If the system is symmetric, the second mode could not be excited by the piezoelectric element and thus the dependency on the parameter $\gamma$. Writing the expression for displacement,

$$W(x) = \frac{H_1(V_{in}-V_{out})\beta^6\ell^2}{\Delta}\left[x^3\left(\frac{\omega^2}{3}\gamma J_0\right) + x^2\left(\frac{6H_2-\ell\omega^2 J_0(1+2\gamma)}{3}\right) + x\left(\frac{\ell^2\omega^2 J_0(1+\gamma)-6H_2\ell}{3}\right)\right] \tag{28}$$

*This point was brought to light in a discussion with W. B. Joyce.

We again observe the dependency of the second mode on $\gamma$.

## III. Experimental Results

Several filters similar to the one in Fig. 1b were fabricated. It is envisioned that the output of the filter will be tied to the gate of an FET, and hence the transfer function was measured with a relatively high terminating impedance. This impedance was low enough to improve the low frequency suppression but high enough as to leave the effective Q of the filter unimpaired.

Ceramic PZT-5, a commercial preparation from Clevite Corp., was used for the piezoelectric elements. For the experimental devices the filter was fabricated from copper, mainly because of ease of fabrication and bonding. A practical device would require a nickel-iron alloy with a suitable combination of temperature coefficient of expansion and Young's modulus. In all the devices $t_d = t_m$. The resonant frequency was near 500 Hz. Typical dimensions are $t_m = 20$ mils, $\ell = 500$ mils, $t_w = 50$ mils. The Q of a typical device was 40, limited primarily by the Q of the PZT. The ratio $C_0/C_1$ was determined by impedance measurements. This ratio was approximately 20. Since $k_{13}^2 = .1$ for PZT, this value is in close agreement with that predicted by Eq. (27) for $u = 1$. A typical transfer function is shown in Fig. 5. The theoretical transfer function is shown in Fig. 6. This function is given for both open circuit conditions and for a termination which in this case has the value $R_L = \frac{5}{\omega_0 C_0}$, $\omega_0 = \frac{1}{\sqrt{CL}}$ where C is the series combination of $C_1$ and $C_0$. The experimental and theoretical plots are in qualitative agreement, if not quite in quantitative agreement. The discrepancy may arise from the arbitrary manner that losses were introduced into the equivalent circuit, and also the arbitrary values assigned to $\gamma$, the asymmetry parameter, and $Q_2$ the quality factor of the parallel resonant circuit. The effect of the couplers has been neglected; their effect is to stiffen the system, i.e. a compliance in series with $L_1$ and $C_1$, raising the resonant frequency. In addition the PZT does not cover the entire web. The effect of the metal extending beyond the PZT and coupled to the weights is a compliance that shunts $L_1$. This additional resonant circuit contributes to the structure of the transfer function at frequencies above the passband. Aside from the shape factors, the experimental and theoretical curves also differ in regard to magnitude. The experimental curves all exhibit the same general shape, but otherwise identical filters may vary widely in output. Figure 7a depicts an equivalent circuit that would explain this situation. The shunt capacitance $C_s$ introduces a voltage divider into the circuit. Physically this shunt capacitance is introduced via the PZT-metal interface. If this interface is rigid then the shunt capacitance is small. If the bond is porous, thick or in any way elastic, the shunt capacitance may become appreciable. The poling electrodes on the PZT were utilized as terminals and were soldered to the metal web. In several cases of high insertion loss, the PZT was removed and rebonded and the result was an improved output.

Since the filter is working into a high impedance termination the output may be improved by using an output transducer smaller than the input transducer, with a resulting voltage transformation.

Two filters coupled together through their supports may be represented by the equivalent circuit in Fig. 7b. These filters can be symmetrically excited, requiring proper poling of the PZT. The value of $C_c$ may be determined from the dimensions and mechanical properties of the coupler. This value in mechanical units is then transformed to electrical units by dividing by the transformation factor $\varphi^2$.(1,3) $\varphi^2$ is derived from the equations of the system; in the present case for $\gamma = 0$, from Eq. (27) we find,

$$\varphi^2 = \frac{\varepsilon^2 B^2 t_m^2 (1+u)^2 \ell_w^2}{2} \tag{29}$$

An experimental transfer function is shown in Fig. 8. Although there is a vast improvement over a single section, as was to be expected, the work on multiple section filters was not pursued further for several reasons. For one to obtain the proper single peaked passband a large coupling compliance was required and the resulting unit was neither compact nor rugged. Furthermore it was much more susceptible to microphonics than the one section filter. In addition tuning the filter was a significantly more difficult task.

In conclusion a one section filter with a Q of 50 to 100 should be sufficient to meet the specifications. The problem of stable materials is discussed in the literature[2,4] and they appear to be available. The tuning of a one section filter is relatively simple particularly since asymmetry can be tolerated. The problems of mounting and microphonics require further study. The experimental filters were mounted in polyfoam and this proved to be a satisfactory arrangement.

J. S. JAYSON

MH-2655-$\frac{JSJ}{JCI}$-KM

J. C. IRWIN

Att.
References 1-4
Figures 1-8

## REFERENCES


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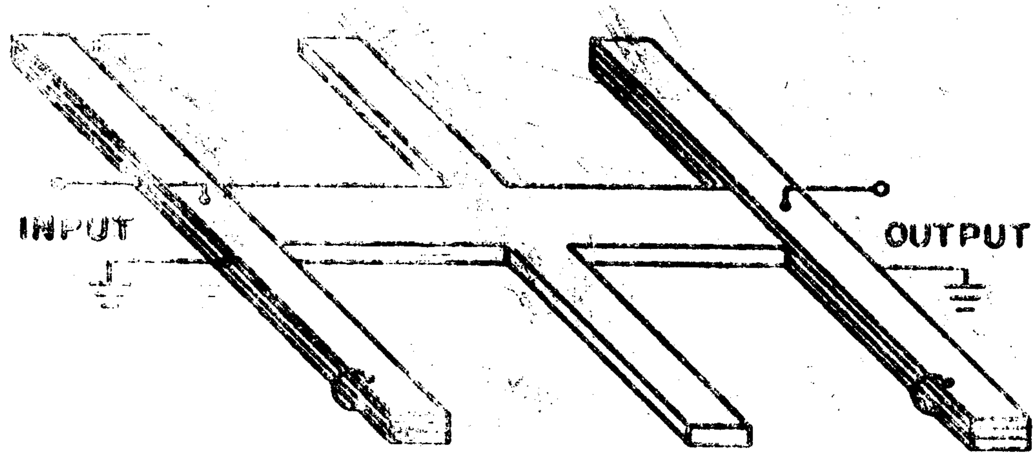
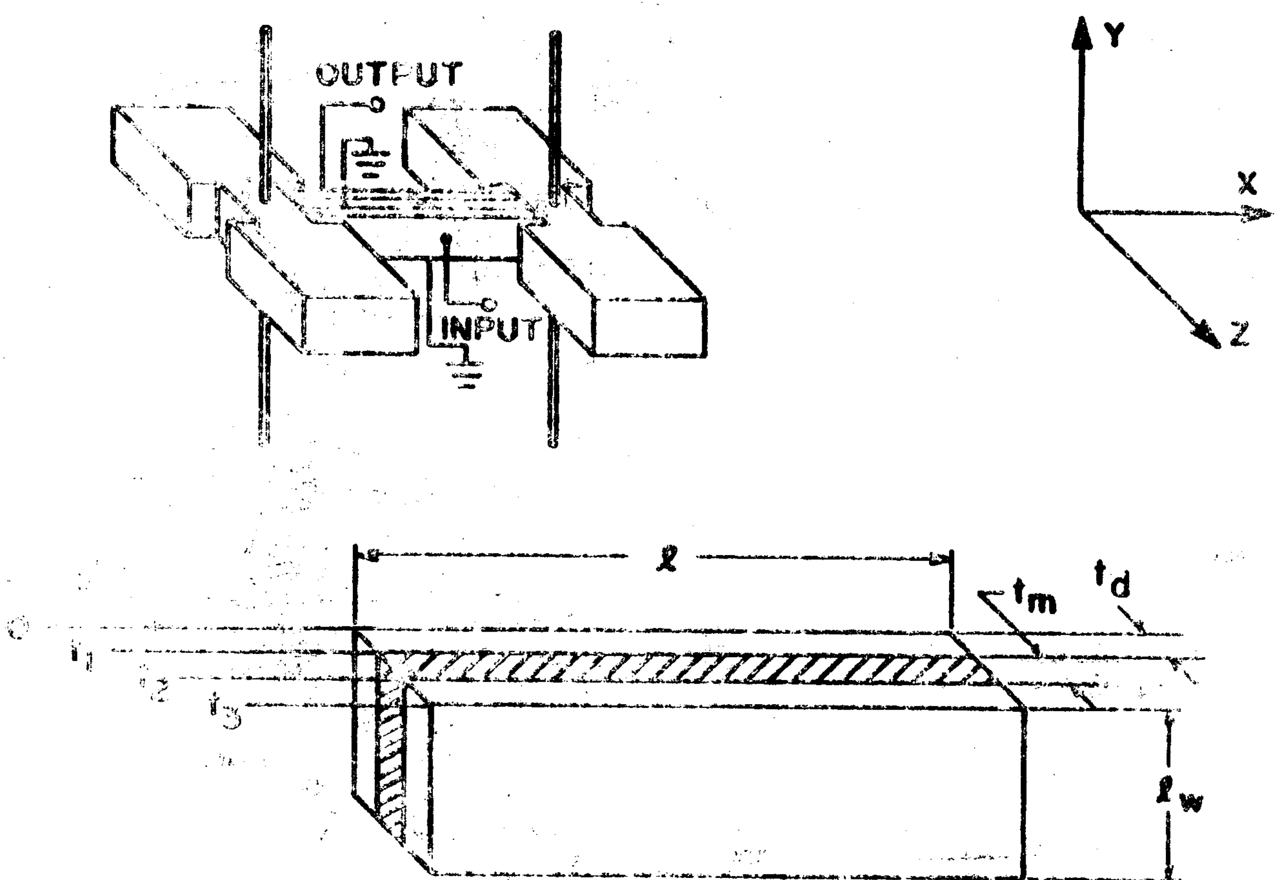
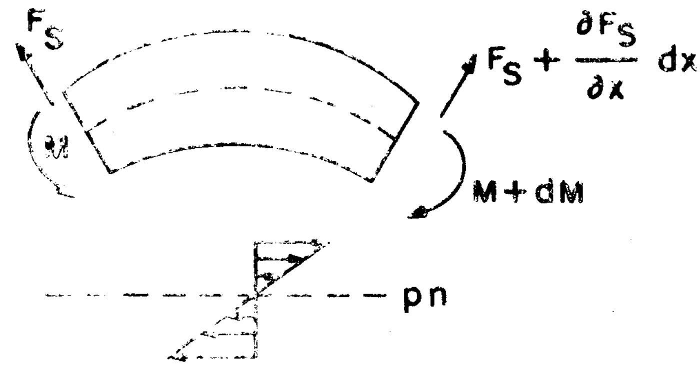



(A) MASON AND THURSTON FILTER

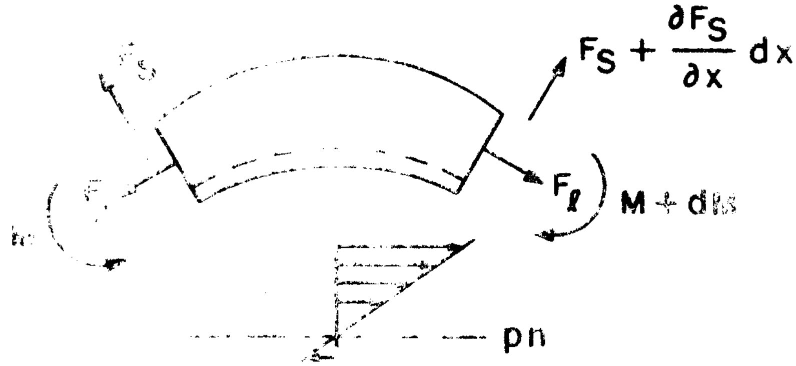



(B) LUMPED CONSTANT FILTER

FIGURE 1

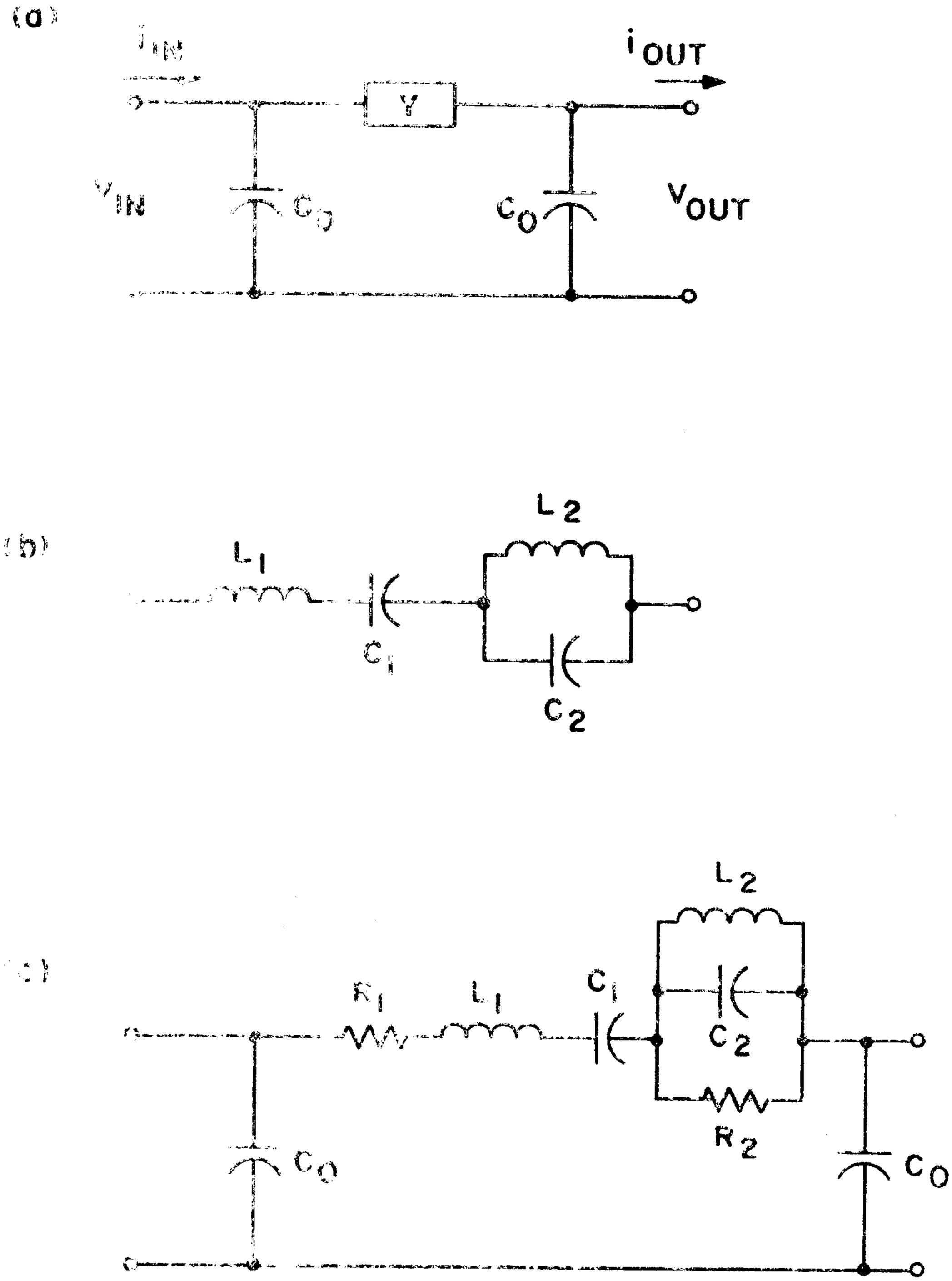



(a) UNIFORM BEAM IN FLEXURE, NO AXIAL FORCE



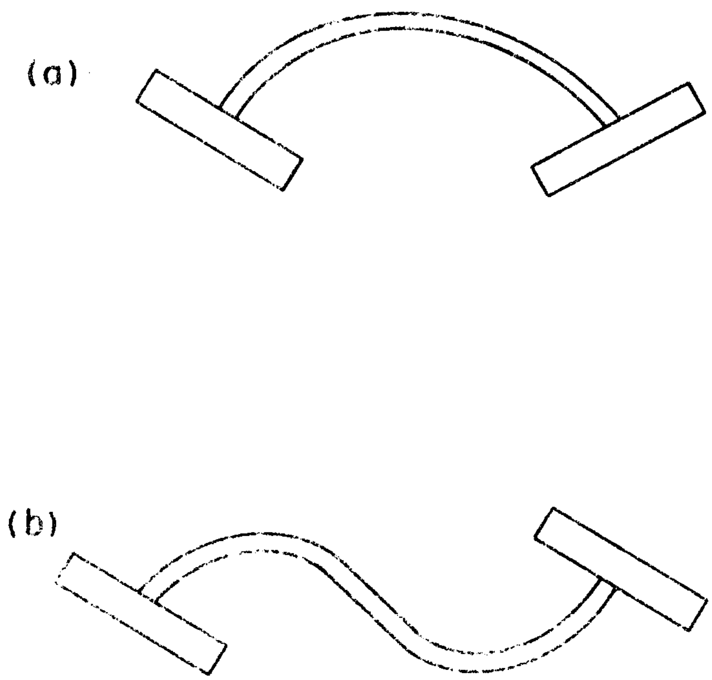

(b) BEAM IN FLEXURE WITH AXIAL FORCE

FIGURE 2



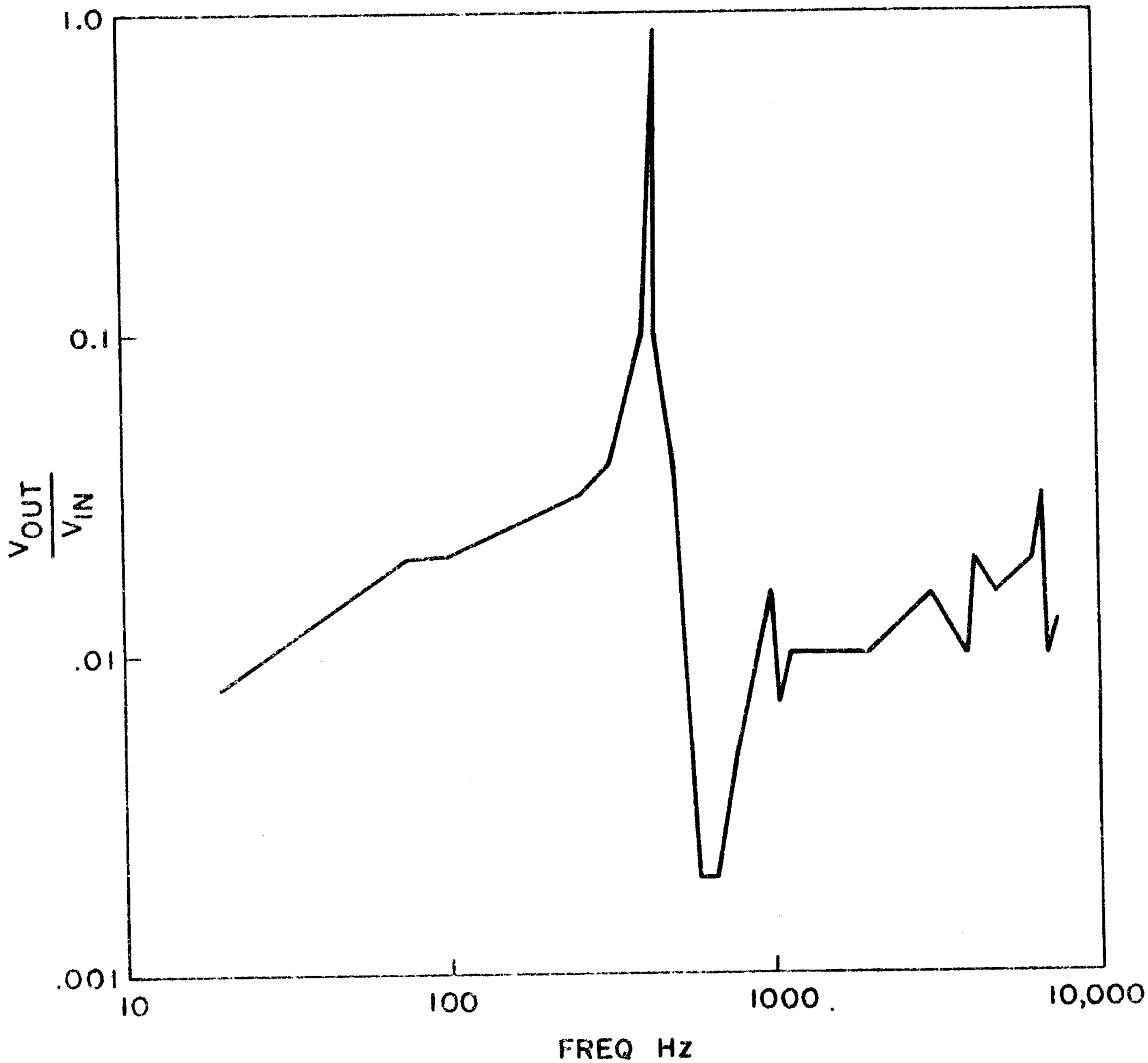

EQUIVALENT CIRCUIT OF ELECTRO-MECHANICAL FILTER

FIGURE 3



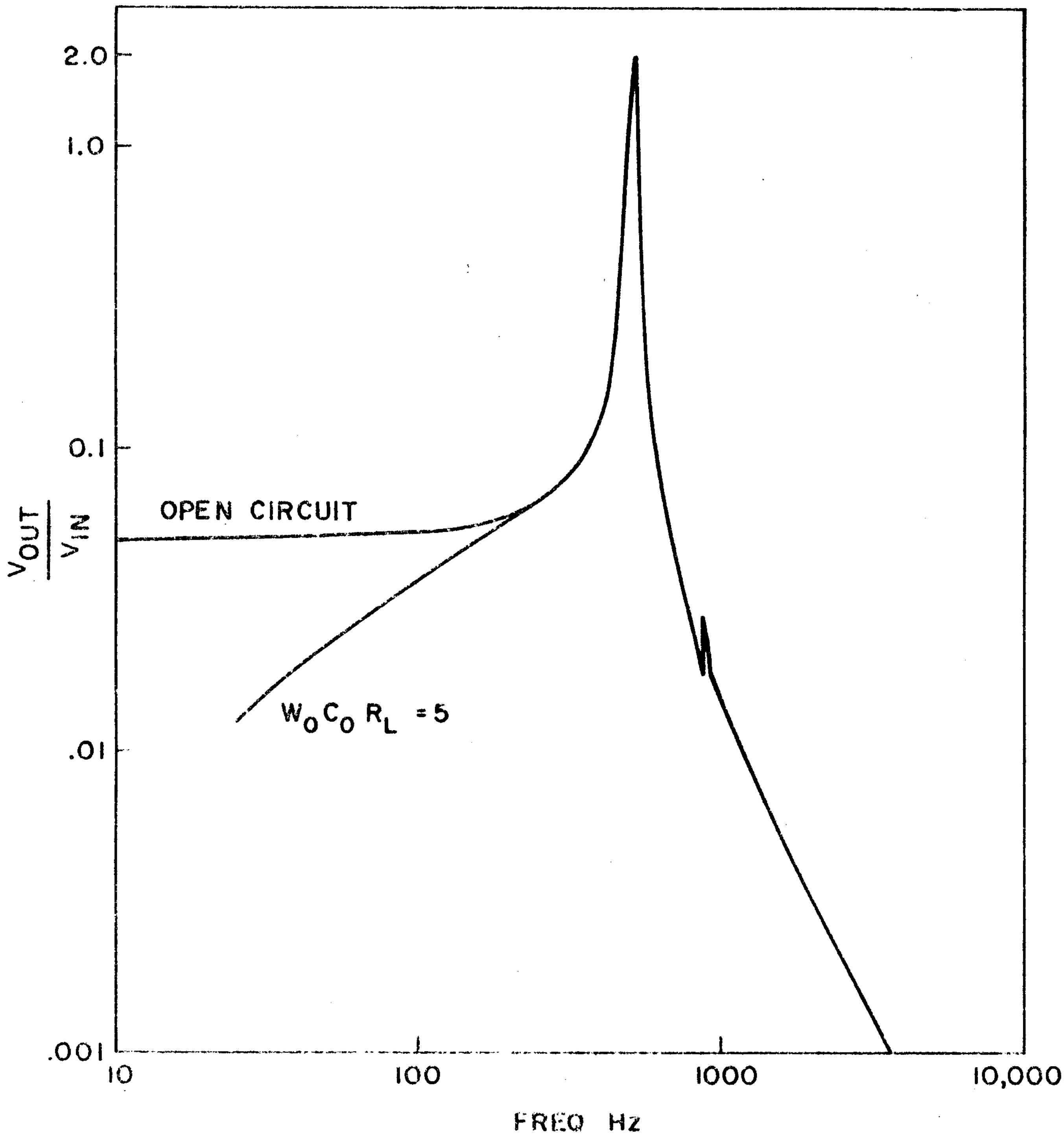

THE TWO LOW FREQUENCY MODES OF FILTER

FIGURE 4



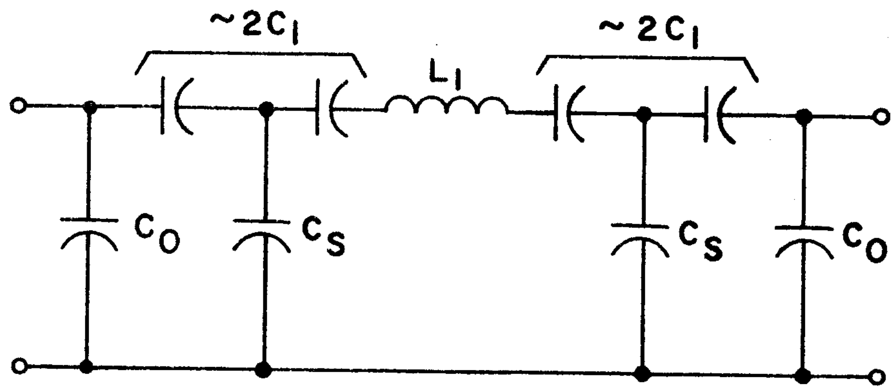

EXPERIMENTAL TRANSFER FUNCTION, SINGLE SECTION FILTER

FIGURE 5

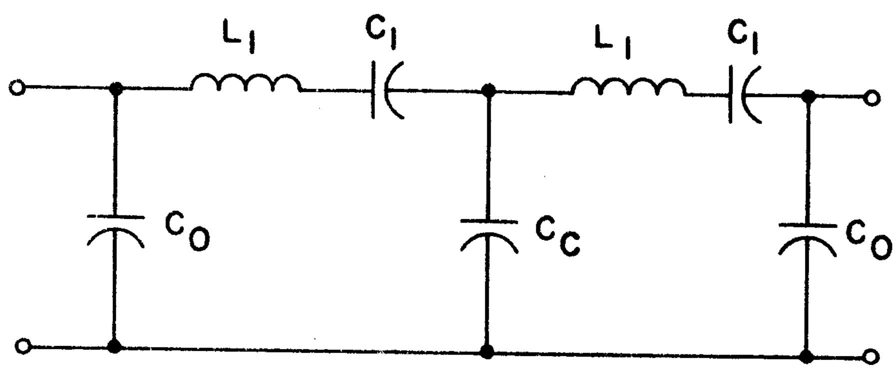



THEORETICAL TRANSFER FUNCTION, Q = 40
$\beta$ = .3

FIGURE 6

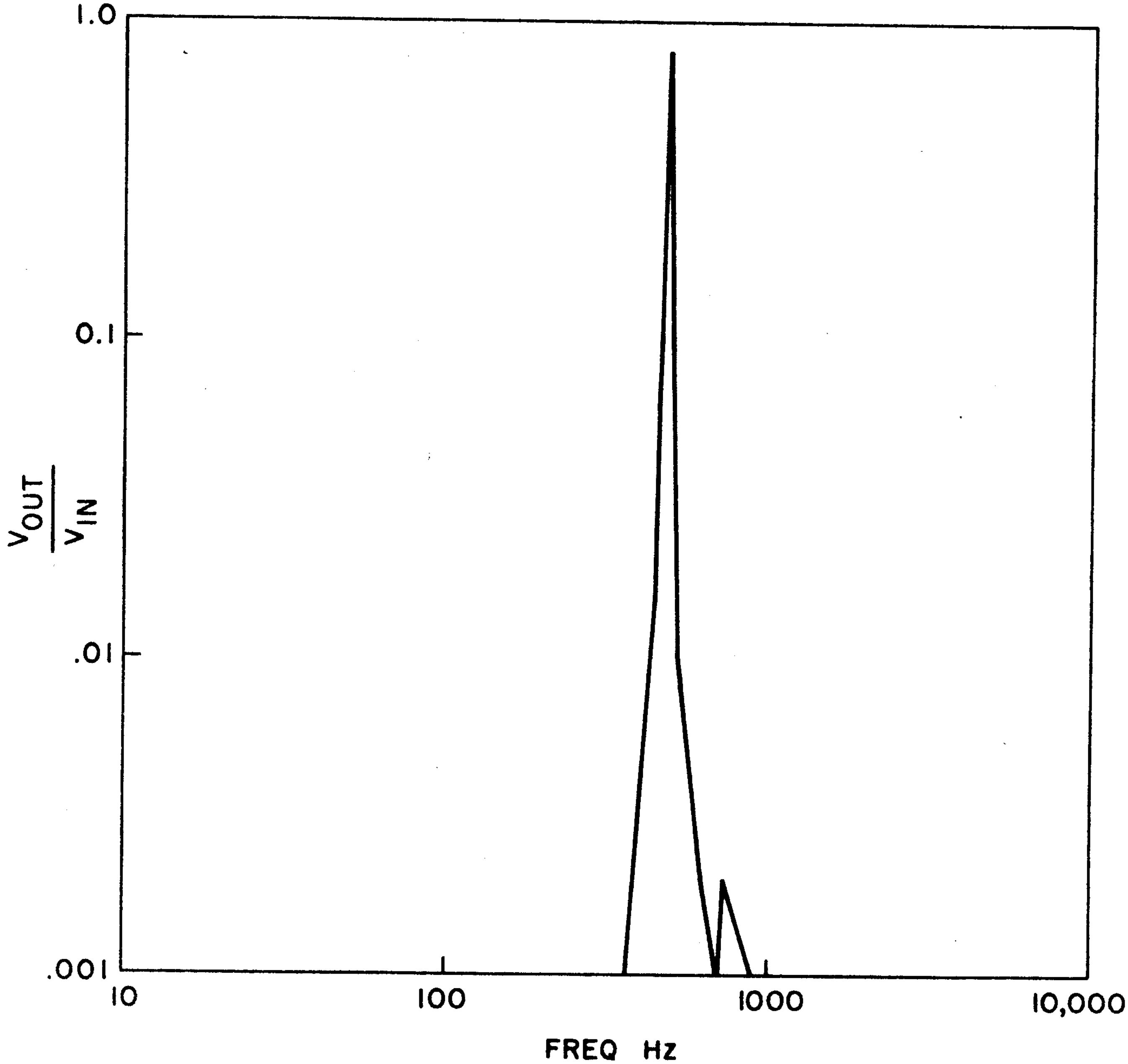



(a) EQUIVALENT CIRCUIT ILLUSTRATING VOLTAGE DROP DUE TO COUPLING OF PIEZOELECTRIC MATERIAL TO FILTER THROUGH PLIANT BOND, $\gamma = 0$




(b) EQUIVALENT CIRCUIT FOR TWO SECTION FILTER, $\gamma = 0$.

FIGURE 7




EXPERIMENTAL TRANSFER FUNCTION, TWO SECTION FILTER

FIGURE 8